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PROPAGATION PROPERTIES OF NONLINEAR OWN WAVES IN A VISCOELASTIC SHELL WITH LIQUID

Teshayev M.KH.1,
Ibragimova D.A.2
1 Chief researcher of the Bukhara branch of the Mathematical Institute.
V.I. Romonovsky, Doctor of Physical and Mathematical Sciences (D.Sc.)
2 Doctoral Students of Samarkand State University

Abstract
The present article deals with further developing of perturbation method for deformation non-linear waves in an elastic cylinder shell, filled with viscous incompressible liquid without inertia of its movement, surrounded by an elastic media and under constraction damping in longitudinal direction. Surrounding medium presence leads to integral-differential equation, to generalizing Korteweg-de Veries ones and possessing the same solution in the form of a solitary wave – a solution. It does not contain an arbitrary constant number unlike Korteweg-de Veries equation solution. The viscous incompressible liquid presence inside the shell behavior is described by means of dynamics and continuity rquation, is solved together with boundary conditions liquid adhesion to a shell wall. The numerical investication is carried out with the use of the modern approach, relying on the universial algorithm of communative algebra for integro-interpolation method. As a result of difference Grobner basis construction, the difference Crank-Nicolson type schemes are generalized. The schemes were obtained due to the use of basic integral difference correlations, approximating the initial equations system. Computational experiment showed that in the case of construction damping and liquid impact have opposite signs but coincide in value, their influence does not case and a solition propagates without changing its direction and its amplitude, which coincide analytical solution. If constructive damping exceeds liquid impact, the wave amplitude decreases, in the opposite case the wave amplitude grouse.

Keywords: nonlinear waves, viscousincompressible liquid, elastic cylinder shell.

The interaction of elastic elements of structures with a liquid was considered in different aspects. The interaction of elastic shells with an ideal incompressible and compressible fluid (gas) was studied in [1, 2]. In an absolutely rigid pipe with a circular cross section laminar motion of a viscous incompressible fluid under the influence of harmonic behavior over the time of the pressure drop was analyzed in [3], and in the case of pulsating motion of a viscous liquids in coaxial elastic shells of finite length, see [4, 5]. Under conditions of vibration, mutual the interaction of a viscous incompressible fluid with elastic shells was studied in [6–9], and with taking into account the rotation of the fluid, see [10–12].

In modern wave dynamics, one of the important areas is the study of the behavior of deformation waves in elastic thin-walled structures. Wave propagation problem in gas dynamics and the theory of elastic shells is studied using linearized equationsny. In this case, the perturbation propagation velocity is assumed to be constant and equal to the velocitypropagation of sound in an undisturbed medium. However, a number of phenomena, despite the small dependent

variables, is entirely determined by the dependence of the propagation velocity perturbations on the magnitude of dependent variables and is studied on the basis of nonlinear equations. These studies are carried out using perturbation methods such as the spliced method. asymptotic expansions, the method of deformable coordinates, the method of multiscale expansions provisions.

For the first time, the Korteweg–de Vries equations for longitudinal nonlinear waves in rods were obtained in [13], and taking into account the dissipative factors, the Korteweg–de Vries–Burgers equations for rods and plates were obtained in [14]. Review of works on the propagation of nonlinear waves and experimental detection of such waves is given in [15].

Deformation waves in elastic cylindrical shells were considered in [16–18]. Except in addition, the problems of wave propagation in elastic and viscoelastic thin-walled structures, including in infinitely long cylindrical shells without interaction with viscous incompressible fluid, were considered in [19–21] from the standpoint of the theory of solitons. Receipt exact solutions of evolutionary equations, including equations for the propagation of solitary waves, considered in [22, 23].

Mathematical models are known that take into account the influence of a viscous incompressible fluid on wave processes in infinitely long geometrically and physically nonlinear shells kah [24–26]. At the same time, the effects of the influence of a viscous incompressible fluid on the behavior of deformation waves in the shell depending on the Poisson's ratio of the shell material. In particular, in the presence of liquid in a shell of inorganic materials (various pipelines in technological facilities), an exponential increase in the wave amplitude was revealed. In the case of organic material (blood vessels), the wave in the liquid quickly decays. The solution of the problem posed in the work for geometrically nonlinear shells is is relevant and complex and is important for acoustic diagnostics and destructive control of materials. In many ways, interest in such problems was initiated by the need the need to analyze the elastic and dynamic properties of nanoobjects, in particular, carbon nanotubes.

In this paper, we develop a perturbation method for modeling nonlinear waves deformations in an elastic cylindrical shell filled with a viscous incompressible fluid, surrounded by an elastic medium, and with structural damping in the longitudinal direction research institutes. The influence of a viscous incompressible fluid filling the shell, surrounding elastic medium and structural damping on the behavior of longitudinal deformation waves in an elastic cylindrical shell.

1. Statement of the problem

Consider an infinitely long elastic cylindrical shell, inside which viscous incompressible fluid. The equations of motion of a viscous incompressible fluid and the continuity equation in the cylindrical coordinate system r, J, x for the case of axisymmetric lateral flow are written as [27]

$$\frac{\partial V_r}{\partial t}+V_r\frac{\partial V_r}{\partial r}+V_x\frac{\partial V_r}{\partial x}+\frac{1}{\rho}\frac{\partial p}{\partial r}=v\left(\frac{\partial^2 V_r}{\partial r^2}+\frac{1}{r}\frac{\partial V_r}{\partial r}+\frac{\partial^2 V_r}{\partial x^2}-\frac{V_r}{r^2}\right),$$

$$\frac{\partial V_x}{\partial t}+V_r\frac{\partial V_x}{\partial r}+V_x\frac{\partial V_x}{\partial x}+\frac{1}{\rho}\frac{\partial p}{\partial x}=v\left(\frac{\partial^2 V_x}{\partial r^2}+\frac{1}{r}\frac{\partial V_x}{\partial r}+\frac{\partial^2 V_x}{\partial x^2}\right), \quad (1)$$

$$\frac{\partial V_r}{\partial r}+\frac{V_r}{r}+\frac{\partial V_x}{\partial x}=0.$$

At the boundary with the shell, the no-slip conditions for the liquid are satisfied according to the approach Lagrange

$$\frac{\partial U}{\partial t}=V_x+U\frac{\partial V_x}{\partial x}-W\frac{\partial V_x}{\partial r},\quad -\frac{\partial W}{\partial t}=V_r+U\frac{\partial V_r}{\partial x}-W\frac{\partial V_r}{\partial r}, \qquad (2)$$

Here $t-$ is time; $V_r,\ V_x$ are projections of the velocity vector on the axes of the cylindrical coordinate system; $p-$ is pressure; $r-$ is the density; $n-$ is the kinematic viscosity coefficient; U - longitudinal elastic moving the shell along the x axis; $W-$ is the deflection of the shell, positive to the center of curvature; $R-$ is the radius of the middle surface of the shell; h_0 is the shell thickness, $(h_0=2(R-R_1))$, where R1 is inner radius of the shell, and $h_0<<R$.

Writing down the equations of motion of an element of a cylindrical shell in displacements for Kirchhoff–Love model, consider a material with a linear dependence of the stress intensity σ_i as a function of strain intensity e_i

$$\sigma_i=Ee_i \qquad (3)$$

(E is Young's modulus).

In addition, we take into account the structural damping in the shell material, which is characterized by which is proportional to $\frac{\partial U}{\partial t}$, added to the d'Alembert inertia force, in pro-longitudinal direction [28]

$$\varepsilon_1\frac{1}{I}\sqrt{\frac{E}{\rho_0(1-\mu_0^2)}}\rho_0h_0\frac{\partial U}{\partial t} \qquad (4)$$

Here ε_1 is the damping coefficient; $\rho_0$is the density of the shell material; μ_0 is the coefficient Poisson; $\sqrt{\frac{E}{\rho_0(1-\mu_0^2)}}=c_0$velocity of propagation of longitudinal waves in the shell; l is the wavelength.

The equations of dynamics of a geometrically nonlinear shell, taking into account (3), (4), are written as

$$\frac{Eh_0}{1-\mu_0^2}\frac{\partial}{\partial x}\left\langle\left\{\frac{\partial U}{\partial x}+\frac{1}{2}\left[\left(\frac{\partial U}{\partial x}\right)^2+\left(\frac{\partial W}{\partial x}\right)^2\right]+\frac{h_0^2}{24}\left(\frac{\partial^2W}{\partial x^2}\right)^2-\mu_0\left(\frac{W}{R}-\frac{1}{2}\frac{W^2}{R^2}-\frac{h_0^2}{24}\frac{W^2}{R^4}\right)\right\}\right\rangle-\rho_0h_0\frac{\partial^2U}{\partial t^2}$$
$$+\varepsilon_1\frac{1}{I}\sqrt{\frac{E}{\rho_0(1-\mu_0^2)}}\rho_0\mu_0\frac{\partial U}{\partial t}-\left[k_3\frac{R^2\rho_0h_0c_0^2}{I^4}U-k_2\frac{\rho_0h_0c_0^2}{R^2I^2}U^3\right]=-q_x-U\frac{\partial q_x}{\partial x}+W\frac{\partial q_x}{\partial r}; \qquad (5)$$

$$\frac{Eh_0}{1-\mu_0^2}\left\langle\frac{h_0^2}{12}\frac{\partial^2}{\partial x^2}\left[\frac{\partial^2W}{\partial x^2}\left(1+\frac{\partial U}{\partial x}\right)+\mu_0\frac{W}{R^2}\left(1-\frac{W}{R}\right)\right]\right.$$
$$-\frac{1}{R}\left\{\mu_0\frac{\partial U}{\partial x}+\frac{1}{2}\mu_0\left[\left(\frac{\partial U}{\partial x}\right)^2+\left(\frac{\partial W}{\partial x}\right)^2\right]+\mu_0\frac{h_0^2}{24}\left(\frac{\partial^2W}{\partial x^2}\right)^2-\left(\frac{W}{R}-\frac{1}{2}\frac{W^2}{R^2}-\frac{h_0^2}{24}\frac{W^2}{R^4}\right)\right\}$$
$$\left.-\frac{\partial}{\partial x}\left\{\frac{\partial W}{\partial x}\left\{\frac{\partial U}{\partial x}+\frac{1}{2}\left[\left(\frac{\partial U}{\partial x}\right)^2+\left(\frac{\partial W}{\partial x}\right)^2\right]+\frac{h_0^2}{24}\left(\frac{\partial^2W}{\partial x^2}\right)^2-\mu_0\left(\frac{W}{R}-\frac{1}{2}\frac{W^2}{R^2}-\frac{h_0^2}{24}\frac{W^2}{R^4}\right)\right\}\right\}\right\rangle$$
$$+\rho_0h_0\frac{\partial^2W}{\partial t^2}+\left[k_1\frac{\rho_0h_0c_0^2}{I^2}W-2t_1\rho_0h_0c_0^2\frac{\partial^2W}{\partial x^2}+m_0\rho_0h_0\frac{\partial^2W}{\partial t^2}\right]$$
$$=q_n+U\frac{\partial q_n}{\partial x}-W\frac{\partial q_n}{\partial r};$$

Here q_x, q_n are the stresses from the side of the liquid inside the shell. Expressions in bracket system (5)

$$-k_3\frac{R^2\rho_0 h_0 c_0^2}{I^4}U + k_2\frac{\rho_0 h_0 c_0^2}{R^2 I^2}U^3;$$

$$k_1\frac{\rho_0 h_0 c_0^2}{I^2}W - 2t_1\rho_0 h_0 c_0^2\frac{\partial^2 W}{\partial x^2} + m_0\rho_0 h_0\frac{\partial^2 W}{\partial t^2} \qquad (6)$$

characterize the Vlasov–Leontiev reaction [29, 30] of the elastic medium in which the pipe is located circular section. Expression $-k_3\frac{R^2\rho_0 h_0 c_0^2}{I^4}U + k_2\frac{\rho_0 h_0 c_0^2}{R^2 I^2}U^3;$

is the response to the longitudinal and the terms $k_1\frac{\rho_0 h_0 c_0^2}{I^2}W$ - reaction to squeezing (compression),

$-2t_1\rho_0 h_0 c_0^2\frac{\partial^2 W}{\partial x^2}$– reaction shift,

$m_0\rho_0 h_0\frac{\partial^2 W}{\partial t^2}$is the inertial reaction.

Dimensionless coefficients of the order of unity k_1, k_2, k_3, t_1, m_0 –are introduced in [29, 30] in dimensional form.

Surface stresses from the side of the liquid carried to the unperturbed middle shell surface (W = R) are determined by the formulas

$$q_x = -\left[\rho v\left(\frac{\partial V_x}{\partial r} + \frac{\partial V_r}{\partial x}\right)\right]_{r=R}, \qquad q_n = \left[-p + 2\rho v\frac{\partial V_r}{\partial r}\right]_{r=R} \qquad (7)$$

2. Derivation of the equation of dynamics, taking into account the presence of liquid in the shell

Taking l (wavelength) as the characteristic length, we pass to dimensionless variables to study equations (5)

$$W = w_m U_3, \quad U = U_m U_1, \quad x^* = \frac{x}{I}, \quad t^* = \frac{c_0}{I}t.$$

Here w_m , $U_m$are the characteristic values of deflection W and longitudinal displacement U. Let us set

$$\frac{h_0}{R} = \varepsilon \ll 1, \quad \frac{R^2}{I^2} = O(\varepsilon), \quad \frac{U_m}{I}\frac{R}{h_0} = O(1), \quad \frac{W_m}{h_0} = O(1),$$

$$\varepsilon_1 = O(\varepsilon), \qquad k_1 = O(1), {}_3 = O(1), {}_2\, t_1 = O(1), \quad m_0 = O(1), \qquad (9)$$

where ε is a small parameter of problem (5).

Let us apply the method of asymptotic expansions, introducing independent variables in the form

$$\xi = x^* - ct^*, \quad \tau = \varepsilon t^* \qquad (10)$$

where $c -$ is the dimensionless unknown wave velocity, t is the fast time, and the dependent variables in the form of expansion in a small parameter ε .

Substituting (8), (10), (11) into equations (5), taking into account estimates (9), we obtain in the zeroth approximation according to ε

$$-\mu_0\frac{\partial U_{10}}{\partial \xi} + \frac{W_m I}{U_m R}U_{30} = 0,$$

$$\frac{\partial^2 U_{10}}{\partial \xi^2} - \mu_0\frac{W_m I}{U_m R}\frac{\partial U_{30}}{\partial} - c^2\frac{\partial^2 U_{10}}{\partial \xi^2} = 0$$

linear system of equations

$$\frac{W_m I}{U_m R} U_{30} = \mu_0 \frac{\partial U_{10}}{\partial \xi}$$

from which the connection follows and the dimensionless wave velocity is determined

$$c^2 = 1 - \mu_0^2$$

From the next approximation in ε, taking into account (12) and (13), an equation is found that is composite, for U_{10}

$$\frac{\partial^2 U_{10}}{\partial \xi \partial \tau} + \frac{U_m}{I\varepsilon}\frac{\sqrt{1-\mu_0^2}}{2}\frac{\partial U_{10}}{\partial \xi}\frac{\partial^2 U_{10}}{\partial \xi^2} + \left\{\frac{1}{\varepsilon}\frac{R^2}{I^2}\frac{\mu_0^2\sqrt{1-\mu_0^2}}{2}[1+m_0] - \frac{\mu_0^2}{\sqrt{1-\mu_0^2}}\frac{t_1}{\varepsilon}\frac{R^2}{I^2}\right\}\frac{\partial^4 U_{10}}{\partial \xi^4}$$
$$+ \frac{\mu_0^2}{2\sqrt{1-\mu_0^2}}\frac{k_1}{\varepsilon}\frac{R^2}{I^2}\frac{\partial^2 U_{10}}{\partial \xi^2} - \frac{k_3}{\varepsilon}\frac{R^2}{I^2}U_{10} + \frac{k_2}{\varepsilon}\frac{U_m^2}{R^2}U_{10}^3 + \frac{1}{2}\frac{\varepsilon_1}{\varepsilon}\frac{\partial U_{10}}{\partial \xi}$$
$$= -\frac{1}{2\sqrt{1-\mu_0^2}}\frac{I^2}{\varepsilon U_0 \rho_0 h_0 c_0^2}\left[q_x - \mu_0 \frac{R}{I}\frac{\partial q_n}{\partial \xi}\right] \qquad (14)$$

In the absence of liquid, the right side of equation (14) is equal to zero and it turns out the well-known equation is the modified Gardner–Burgers–Ostrovsky equation for $\frac{\partial U_{10}}{\partial} = \frac{1}{\mu_0}\frac{w_m I}{U_m R}U_{30}$.

3. Determination of stresses acting on the liquid side

To determine the right side of equation (14), we introduce dimensionless variables and parameter

$$V_r = w_m \frac{c_0}{I} V_r, V_x = w_m \frac{c_0}{R_1} V_x, r^* = \frac{r}{R}, p = \frac{\rho v c_0 I w_m}{R^3} P, \psi = \frac{R}{I} = O\left(\varepsilon^{\frac{1}{2}}\right) \quad (15)$$

Substituting (15) into the equation of hydrodynamics (1) and boundary conditions (2), we represent the indifferent measured velocities and pressure in the form of an expansion in terms of the small parameter ε

$$V_x = V_x^0 + \varepsilon V_x^1 + \cdots, V_r = V_r^0 + \varepsilon V_r^1 + \cdots, P = P^0 + \varepsilon P^1 + \cdots \quad (16)$$

In the zeroth approximation in ψ (ψ=0is the hydrodynamic theory of lubrication), in the zero approximation with respect to ε we obtain the equations of the rhodynamics (classical equations of the hydrodynamic theory of lubrication)

$$\frac{\partial P^0}{\partial r^*} = 0;\ \psi \frac{R_1 c_0}{v}\frac{\partial V_x^0}{\partial t^*} + \frac{\partial P^0}{\partial x^*} = \frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^*\frac{\partial V_x^0}{\partial r^*}\right);\ \frac{1}{r^*}\frac{\partial}{\partial r^*}(r^* V_x^0) + \frac{\partial V_x^0}{\partial x^*} == 0 \quad (17)$$

and boundary conditions

$$V_r^0 = -\frac{\partial U_3}{\partial t^*};\ V_x^0 = \frac{U_m R_1}{w_m I}\frac{\partial U_1}{\partial t^*}, \qquad \text{где } r^* = 1 \qquad (18)$$

From the solution of problem (17), (18) it follows that

$$r^*\frac{\partial V_r^0}{\partial r^*} = 0;\ r^*\frac{\partial V_x^0}{\partial r^*} = 0, \quad \text{где } r^* = 0 \qquad (19)$$

Taking into account that variables (10), (11) were introduced, and having relations (12), (13), from (19) semichim

$$\frac{\partial P^0}{\partial r^*} = 0;\ \frac{\partial P^0}{\partial x^*} = \frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^*\frac{\partial V_x^0}{\partial r^*}\right);\ \frac{1}{r^*}\frac{\partial}{\partial r^*}(r^* V_x^0) + \frac{\partial V_x^0}{\partial x^*} = 0 \qquad (20)$$

With the accepted accuracy in ε, ψ from (7) we find

$$q_x = \varepsilon \frac{v}{Rc_0}\rho c_0^2 \left.\frac{\partial V_x}{\partial r^*}\right|_{r^*=1}, \quad q_n = -\frac{\varepsilon}{\psi}\frac{v}{R_1 c_0}\rho c_0^2 P^0 \qquad (21)$$

Substituting (21) into equation (14), we finally obtain

$$\frac{\partial^2 U_{10}}{\partial\xi\partial\tau} + \frac{U_m}{I\varepsilon}\frac{\sqrt{1-\mu_0^2}}{2}\frac{\partial U_{10}}{\partial\xi}\frac{\partial^2 U_{10}}{\partial\xi^2} + \left\{\frac{1}{\varepsilon}\frac{R^2}{I^2}\frac{\mu_0^2\sqrt{1-\mu_0^2}}{2}[1+m_0] - \frac{\mu_0^2}{\sqrt{1-\mu_0^2}}\frac{t_1}{\varepsilon}\frac{R^2}{I^2}\right\}\frac{\partial^4 U_{10}}{\partial\xi^4}$$
$$+ \frac{\mu_0^2}{2\sqrt{1-\mu_0^2}}\frac{k_1}{\varepsilon}\frac{R^2}{I^2}\frac{\partial^2 U_{10}}{\partial\xi^2} - \frac{k_3}{\varepsilon}\frac{R^2}{I^2}U_{10} + \frac{k_2}{\varepsilon}\frac{U_m^2}{R^2}U_{10}^3 + \frac{1}{2}\frac{\varepsilon_1}{\varepsilon}\frac{\partial U_{10}}{\partial\xi}$$
$$= 2\frac{\rho I}{\rho_0 h_0}\frac{1}{\varepsilon}\frac{v}{Rc_0}[1-4\mu_0^2]\frac{\partial U_{10}}{\partial\xi} \qquad (22)$$

Replacement

$$\frac{\partial U_{10}}{\partial\xi} = c_1\varphi, \quad \eta = c_2\xi, \quad \tilde{t} = c_3\tau \qquad (23)$$

allows us to write equation (22) in the form

$$\varphi_{\tilde{t}} + \varphi\varphi_\eta + \varphi_{\eta\eta\eta} + s_2\varphi_\eta + (s_3 - s)\varphi - \int \varphi d\eta + s_5\left(\int \varphi d\eta\right)^3 = 0.$$

The constants c_1, c_2, c_3 are determined by substituting (23) into (22) and have the form

$$c_1 = \sigma_0^{-1}\sigma_4^{1/2}\sigma_1^{1/2}, \quad c_2 = \sigma_4^{1/4}\sigma_1^{-1/4}, \quad c_3 = \sigma_4^{3/4}\sigma_1^{1/4}$$

while introducing the designations

$$s_2 = \sigma_2 c_2 c_3^{-1}, \qquad s_3 = \sigma_3 c_3^{-1}, \qquad s = \sigma c_3^{-1}, \quad s_5 = \sigma_5 c_1^2 c_2^{-3} c_3^{-1},$$

where

Note that in the absence of structural damping, we have $s_3 = 0, s > 0$ for $\mu_0 < 1/2$ for inorganic materials, s < 0 for $\mu_0 > 1/2$ for living organisms and s = 0 with $\mu_0 = 1/2$ for an incompressible material such as rubber or in the absence of liquid.

Note that the case $s_3 = s$ is equivalent to the absence of the influence of the liquid inside the shell.

In the absence of structural damping and liquid or for the case $s_3 = s$ we obtain from (24) the equation

which has the exact solution

$$\varphi = \frac{1}{2s_5}\cosh^{-2}\left\{\frac{1}{2\sqrt{s_5}}\left[\eta - \left(\frac{1}{s_5} + 2s_5 + s_2 - s_6\right)\tilde{t}\right]\right\} \qquad (26)$$

In the absence of a surrounding elastic medium, Eq. (25) turns into the equation Korteweg–de Vries with exact solution

$$\varphi(\eta, 0) = \frac{1}{2s_5}\cosh^{-2}\left\{\frac{1}{2\sqrt{s_5}}\left[\eta - \left(\frac{1}{s_5}\right)t\right]\right\} \qquad (27)$$

for an arbitrary value of s5.

As a result, we obtain the following difference scheme for equation (24), similar to Crank–Nicolson scheme for the heat equation

$$\frac{U_j^{n+1} - U_j^n}{\tau} + 3\frac{\left(U_{j+1}^{2n+1} - U_{j-1}^{2n+1}\right) + \left(U_{j+1}^{2n} - U_{j-1}^{2n}\right)}{4h} +$$

$$+\left(\left(U_{j+2}^{n+1}-2U_{j+1}^{n}+2U_{j-1}^{n}-U_{j-22}^{n+1}\right)+\left(U_{j+2}^{n}-2U_{j+1}^{n}+2U_{j-1}^{n}-U_{j-2}^{n}\right)\right)\Big/4h^{3}+$$

$$(s_2-s_6)\frac{\left(U_{j+1}^{n+1}-U_{j-1}^{n+1}\right)+\left(U_{j+1}^{n}-U_{j-1}^{n}\right)}{4h} \quad (28)$$

$$+(s_3+s)\frac{U_j^{n+1}+U_j^{n}}{2}$$

$$-\frac{U_j^{n+1}+U_j^{n}}{2}+s_5\frac{U_j^{3n+1}+U_j^{3n}}{2}=0,$$

$$\frac{U_{j+2}^{n}-2U_{j+1}^{n}+U_j^{n}}{h^2}-\frac{U_{j+2}^{n}-U_j^{n}}{2h}=0$$

The number of iterations to achieve an accuracy of 10-12 on the next time layer is usually lo, no more than two or three. The time step t was taken equal to half the step in the variable h. The calculation program was written in Python using the SciPy package (http://scipy.org).

Equations (24) numerical solution graphs under$s_3 = 0{:}5,\ s = 1{:}0, s_2 = 1{:}0\ and\ s_5 = 0{:}8$ with the initial condition taken from the exact solution (26) under $t = 0$

Conclusion

In the presented work, a study of the model of wave phenomena in a nonlinear linear elastic cylindrical shell of the Kirchhoff–Love type surrounded by an elastic medium and containing a viscous incompressible fluid. The performed computational experiments, poswanted to evaluate the effect of a viscous incompressible fluid and structural damping on propagation of nonlinear waves in an elastic shell surrounded by an elastic medium. Availability elastic medium leads to an increase in the wave speed.

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