CHEGARAVIY SHARTIDA UMUMLASHGAN KASR TARTIBLI INTEGRODIFFERENSIAL OPERATOR QATNASHGAN CHEGARAVIY MASALA

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Faraz qilaylik, $\Omega - x$ va y o’zgaruvchilar tekisligidagi chekli, bir qiymatli soha bo’lib, $y>0$ bo’lganda

$$|y|^m u_{xx} - u_{yy} = 0 \qquad (m = const > 0) \quad (1)$$

tenglamaning

$$OC_1: x - \frac{2}{m+2} y^{\frac{m+2}{2}} = 0, \quad AC_1: x + \frac{2}{m+2} y^{\frac{m+2}{2}} = 1$$

xarakteristikalari, $y<0$ bo’lganda esa $(-y)^m u_{xx} - u_{yy} = 0$ tenglamaning

$$OC_2: x - \frac{2}{m+2}(-y)^{\frac{m+2}{2}} = 0, \quad AC_2: x + \frac{2}{m+2}(-y)^{\frac{m+2}{2}} = 1$$

xarakteristikalari bilan chegaralangan.

Quyidagi belgilashlarni kiritamiz:

$$\Omega_1 = \Omega \bigcap (y>0), \Omega_2 = \Omega \bigcap (y<0),$$

$$J = \{(x,y): 0<x<l, y=0\}, 2\beta = m/(m+2)$$

$$\theta_1(x) = \frac{x}{2} + i\left(\frac{x(m+2)}{4}\right)^{\frac{2}{m+2}}, \theta_2(x) = \frac{x}{2} - i\left(\frac{x(m+2)}{4}\right)^{\frac{2}{m+2}}$$

bu yerda $\theta_1(x)$ va $\theta_2(x)$ lar (1) tenglamaning $(x,0)\in J$ nuqtadan chiquvchi xarakteristikalari bilan OC_1 va OC_2 xarakteristikalarning kesishish nuqtalari koordinatalari.

(1) tenglamaning $C^1\left(\overline{\Omega}\right)\cap C^2\left(\Omega_1\cup\Omega_2\right)$ sinfga tegishli bo'lib, Ω_1 va Ω_2 sohada bu tenglamani qanoatlantiruvchi $u(x,y)$ yechimi (1) tenglamaning Ω sohadagi regulyar yechimi deyiladi.

Masala. (1) tenglamaning Ω sohada regulyar bo'lib,

$$F_{0x}\begin{bmatrix} a, & b \\ c, & g(x)\end{bmatrix}U\left[\theta_j(x)\right]=a_j(x)U(x,0)+b_j(x) \quad (2)$$

shartni qanoatlantiruvchi $u(x,y)$ funksiya topilsin. Bu yerda a,b,c–haqiqiy sonlar $g(x)=x^2,\, a_1(x),\, a_2(x),\, b_1(x),\, b_2(x)$–berilgan funksiyalar.

$F\begin{bmatrix} a, & b \\ c, & g(x)\end{bmatrix}$–funksiyadan boshqa funksiya bo'yicha olingan integrodifferensial operator [19].

Quyidagi teorema o'rinli bo'ladi.

Teorema. Agar $a_1(x),a_2(x),b_1(x),b_2(x)\in C^3(\overline{J})$ va $a_1(x)\neq a_2(x)$, $-1<\gamma+\beta<0$, $c-a-b>0$ bo'lsa, u holda masalaning yagona yechimi mavjud bo'ladi.

Isbot. (1) tenglamaning $\Omega_1\cup\Omega_2$ sohadagi

$$u(x,0)=\tau\left(x\right),u_y(x,0)=\nu(x),\ (x,0)\in J$$

shartlarni qanoatlantiruvchi yechimi quyidagi ko'rinishda bo'ladi [14].

$$u(x,y)=\frac{\Gamma(2\beta)}{\Gamma^2(\beta)}\int_0^1\tau\left[x+\frac{2}{m+2}y^{\frac{m+2}{2}}(2t-1)\right]\left[t(1-t)\right]^{\beta-1}dt+$$

$$+\frac{\Gamma(2-2\beta)}{\Gamma^2(1-\beta)}y\int_0^1\nu\left[x+\frac{2}{m+2}y^{\frac{m+2}{2}}(2t-1)\right]\left[t(1-t)\right]^{-\beta}dt,(3)$$

va

$$u(x,y)=\frac{\Gamma(2\beta)}{\Gamma^2(\beta)}\int_0^1\tau\left[x+\frac{2}{m+2}(-y)^{\frac{m+2}{2}}(2t-1)\right]\left[t(1-t)\right]^{\beta-1}dt+$$

$$+\frac{\Gamma(2-2\beta)}{\Gamma^2(1-\beta)}y\int_0^1\nu\left[x+\frac{2}{m+2}(-y)^{\frac{m+2}{2}}(2t-1)\right]\left[t(1-t)\right]^{-\beta}dt.(4)$$

Endi $u[\theta_1(x)]$ ni hisoblaymiz.

Agar (3) yechim formulasidan foydalansak,

$$u[\theta_1(x)] = \frac{\Gamma(2\beta)}{\Gamma^2(\beta)}\int_0^1 \tau\left[\frac{x}{2} + \frac{2}{m+2}\frac{m+2}{4}x(2t-1)\right]\left[t(1-t)\right]^{\beta-1}dt +$$

$$+\frac{\Gamma(2-2\beta)}{\Gamma^2(1-\beta)}\left[\frac{m+2}{4}x\right]^{\frac{2}{m+2}}\int_0^1 \nu\left[\frac{x}{2} + \frac{2}{m+2}\frac{m+2}{4}x(2t-1)\right]\left[t(1-t)\right]^{-\beta}dt =$$

$$= \frac{\Gamma(2\beta)}{\Gamma^2(\beta)}\int_0^1 \tau(xt)\left[t(1-t)\right]^{\beta-1}dt + \frac{\Gamma(2-2\beta)}{\Gamma^2(1-\beta)}\left(\frac{m+2}{4}\right)^{1-2\beta} x^{1-2\beta}\int_0^1 \nu(xt)\left[t(1-t)\right]^{-\beta}dt$$

Agar oxirgi integralda $z = xt$ almashtirish bajarsak , u holda

$$u[\theta_1(x)] = \frac{\Gamma(2\beta)}{\Gamma(\beta)}x^{1-2\beta}D_{0,x}^{-\beta}x^{\beta-1}\tau(x) + \left(\frac{m+2}{4}\right)^{1-2\beta}\frac{\Gamma(2-2\beta)}{\Gamma(1-\beta)}D_{0,x}^{\beta-1}x^{-\beta}\nu(x) \quad (5)$$

Xuddi shunga o'xshash $u[\theta_2(x)]$ ni hisoblab,

$$u[\theta_2(x)] = \frac{\Gamma(2\beta)}{\Gamma(\beta)}x^{1-2\beta}D_{0,x}^{-\beta}x^{\beta-1}\tau(x) - \left(\frac{m+2}{4}\right)^{1-2\beta}\frac{\Gamma(2-2\beta)}{\Gamma(1-\beta)}D_{0,x}^{\beta-1}x^{-\beta}\nu(x) \quad (6)$$

tenglikni olamiz. Bu yerda $\Gamma(z)-$Eylerning gamma-funksiyasi.

$u[\theta_1(x)]$ va $u[\theta_2(x)]$ larni ifodalarini (2) chegaraviy shartga qo'yib

$$\frac{\Gamma(2\beta)}{\Gamma(\beta)}F_{0x}\begin{bmatrix}\alpha,\beta\\ \gamma\ x^2\end{bmatrix}x^{1-2\beta}D_{0x}^{-\beta}x^{\beta-1}\tau(x) - \frac{\Gamma(2-2\beta)}{\Gamma(1-\beta)}\left(\frac{m+2}{4}\right)^{1-2\beta}F_{0x}\begin{bmatrix}\alpha,\beta\\ \gamma\ x^2\end{bmatrix}D_{0x}^{\beta-1}x^{-\beta}\nu(x) =$$

$$a_2(x)\nu(x) + b_2(x), \quad (x,0)\in J\,;$$

$$\frac{\Gamma(2\beta)}{\Gamma(\beta)}F_{0x}\begin{bmatrix}\alpha,\beta\\ \gamma, x^2\end{bmatrix}x^{1-2\beta}D_{0x}^{-\beta}x^{\beta-1}\tau(x) + \frac{\Gamma(2-2\beta)}{\Gamma(1-\beta)}\left(\frac{m+2}{4}\right)^{1-2\beta}F_{0x}\begin{bmatrix}\alpha,\beta\\ \gamma, x^2\end{bmatrix}D_{0x}^{\beta-1}x^{-\beta}\nu(x) =$$

$$a_1(x)\nu(x) + b_1(x), \quad (x,0)\in J$$

ifodalarni hosil qilamiz.

Bundan esa

$$\frac{\Gamma(2\beta)}{\Gamma(\beta)}J_1(x)\pm\left(\frac{m+2}{4}\right)^{1-2\beta}\frac{\Gamma(2-2\beta)}{\Gamma(1-\beta)}J_2(x)=a_j(x)\nu\left(x\right)+b_j(x),\ \ j=1,2\ (7)$$

ifodaga ega bo'lamiz. Bu yerda

$$J_1(x)=F_{0x}\begin{bmatrix}\alpha,\beta\\ \gamma,x\end{bmatrix}x^{1-2\beta}D_{0x}^{-\beta}x^{\beta-1}\bar{\tau}(x),$$
$$J_2(x)=F_{0x}\begin{bmatrix}\alpha,\beta\\ \gamma,x\end{bmatrix}D_{0x}^{\beta-1}\sqrt{x}^{-\beta}\bar{\nu}(x),\qquad \bar{\tau}(x)=\tau(\sqrt{x}),\ \bar{\nu}(x)=\nu(\sqrt{x}).$$

Ko'rsatish mumkinki,

$$x^{l_1}F_{0x}\begin{bmatrix}a_1, & b_1\\ c_1, & x\end{bmatrix}\left(\sqrt{x}\right)^{l_2}F_{0x}\begin{bmatrix}a_2, & b_2\\ c_2, & x\end{bmatrix}\phi\left(\sqrt{x}\right)=2^{2-c_2}\left(x^2\right)^{l_1+c_1}\int_0^x\left(y^2\right)^{l_2+c_2}\phi(y)\times$$
$$\times G_{66}^{06}\left(\frac{x^2}{y^2}\left|\begin{matrix}0, & a_1+b_1-c_1, & \frac{1+c_2+l_2}{2}, & \frac{c_2+l_2}{2}, & \frac{1+a_2+b_2+l_2}{2}, & \frac{a_2+b_2+l_2}{2}\\ a_1-c_1, & b_1-c_1, & \frac{1+a_2+l_2}{2}, & \frac{a_2+l_2}{2}, & \frac{1+b_2+l_2}{2}, & \frac{b_2+l_2}{2}\end{matrix}\right.\right)dy.$$

Isbot. Yuqoridagi formulalardan foydalansak,

$$J_1(x)=2^{2-\beta}x^{\gamma}\int_0^x y^{1-2\beta+c}\left(\frac{y}{x}\right)^{\frac{1+3\beta}{2}}y^{\beta-1}\tau(y)\times G_{44}^{40}\left(\frac{y}{x}\left|\begin{matrix}\frac{1-3\beta}{2}-\alpha+\gamma, & \frac{1-5\beta}{2}+\gamma, & -\frac{\beta+1}{2}, & -\frac{\beta}{2}\\ \frac{1-3\beta}{2}, & \frac{1-5\beta}{2}-\alpha+\gamma, & -\frac{1}{2}, & 0\end{matrix}\right.\right)dy=$$ va

$$=2^{2-\beta}x^{\gamma-\frac{1+3\beta}{2}}\int_0^x y^{\frac{3-\beta}{2}+c}y^{\beta-1}\bar{\tau}(y)\times G_{44}^{40}\left(\frac{y}{x}\left|\begin{matrix}\frac{1-3\beta}{2}-\alpha+\gamma, & \frac{1-5\beta}{2}+\gamma, & -\frac{\beta+1}{2}, & -\frac{\beta}{2}\\ \frac{1-3\beta}{2}, & \frac{1-5\beta}{2}-\alpha+\gamma, & -\frac{1}{2}, & 0\end{matrix}\right.\right)dy$$

$$J_2(x)=2^{2-\beta}x^{\gamma}\int_0^x y^{1-\beta}\sqrt{y}^{-\beta}\bar{\nu}(y)\times G_{33}^{30}\left(\frac{y}{x}\left|\begin{matrix}1-\alpha+\gamma, & 1-\beta+\gamma, & \frac{1}{2}\\ 1-\alpha-\beta+\gamma, & \frac{2-\beta}{2}, & \frac{3-\beta}{2}\end{matrix}\right.\right)dy.$$

Agar yuqoridagi tengliklarni inobatga olsak,

$$\frac{\Gamma(2\beta)}{\Gamma(\beta)}J_1(x)-\frac{\Gamma(2-2\beta)}{\Gamma(1-\beta)}\left(\frac{m+2}{4}\right)^{1-2\beta}J_2(x)=\bar{a}_2(x)\bar{\nu}(x)+\bar{b}_2(x)\ (8)$$

$$\frac{\Gamma(2\beta)}{\Gamma(\beta)}J_1(x)+\frac{\Gamma(2-2\beta)}{\Gamma(1-\beta)}\left(\frac{m+2}{4}\right)^{1-2\beta}J_2(x)=\bar{a}_1(x)\bar{\nu}(x)+\bar{b}_1(x)\ (9)$$

ifodalarni hosil qilamiz.

Agar (8) va (9) tengliklarni hadma-had qo'shsak,

$$2\frac{\Gamma(2\beta)}{\Gamma(\beta)}J_1(x)=\left(\bar{a}_1(x)+\bar{a}_2(x)\right)\bar{\nu}(x)+\left(\bar{b}_1(x)+\bar{b}_2(x)\right)$$

$$2\frac{\Gamma(2\beta)}{\Gamma(\beta)}J_1(x)=(a_1(x)+a_2(x))\nu(x)+(b_1(x)+b_2(x)). \quad (10)$$

(9) tenglikdan (8) tenglikni ayirib,

$$2\frac{\Gamma(2-2\beta)}{\Gamma(1-\beta)}\left(\frac{m+2}{4}\right)^{1-2\beta}J_2(x)=\left(a_1(x)-a_2(x)\right)\nu(x)+b_1(x)-b_2(x) \text{ yoki}$$

$$2\frac{\Gamma(2-2\beta)}{\Gamma(1-\beta)}\left(\frac{m+2}{4}\right)^{1-2\beta}2^{2-\beta}x^{\gamma}\int_0^x y^{1-\beta}\sqrt{y}^{-\beta}\bar{\nu}(y)\times G_{33}^{30}\left(\frac{y}{x}\middle|\begin{matrix}1-\alpha+\gamma, & 1-\beta+\gamma, & \frac{1}{2}\\ 1-\alpha-\beta+\gamma, & \frac{2-\beta}{2}, & \frac{3-\beta}{2}\end{matrix}\right)dy=$$

$$=\left(\bar{a}_1(x)-\bar{a}_2(x)\right)\bar{\nu}(x)+\bar{b}_1(x)-\bar{b}_2(x) \quad (11)$$

tengliklar kelib chiqadi.

(11) tenglikdan ba'zi almashtirishlardan so'ng

$$\bar{\nu}(x)-\lambda\int_0^x \mathrm{K}(x,t)\bar{\nu}(x)dt=F(x) \quad (12)$$

integral tenglamani hosil qilamiz.

Bu yerda

$$\lambda=2^{2-\beta}\frac{\Gamma(2-2\beta)}{\Gamma(1-\beta)}\left(\frac{m+2}{4}\right)^{1-2\beta},$$

$$\mathrm{K}(x,t)=G_{44}^{04}\left(\frac{x^2}{y^2}\middle|\begin{matrix}0, & \alpha+\beta-\gamma, & \frac{2-\beta}{2}, & \frac{1-\beta}{2}\\ \alpha-\gamma, & \beta-\gamma, & \frac{1}{2}, & 0\end{matrix}\right)\cdot y^{1-\beta}\frac{2}{a_1(y)-a_2(y)}, \quad (13)$$

$$F(x,t)=\frac{b_2(x)-b_1(x)}{a_1(x)-a_2(x)}$$

hosil bo'lgan (12) tenglama Volterra tipidagi 2-tur integral tenglamadir. Bu tenglama yagona yechimga ega bo'ladi.

(12) tenglikning har ikki tomoniga

$$\frac{d}{dx}x^{-\frac{\beta}{2}-1}\int_0^x G_{44}^{40}\left(\frac{y}{x}\left|\begin{matrix}\frac{1-2\beta}{2}, & \frac{1-4\beta}{2}-\alpha+\gamma, & \frac{\beta-1}{2}, & \frac{2+\beta}{2}\\ \frac{1-2\beta}{2}-\alpha+\gamma, & \frac{1-4\beta}{2}+\gamma, & -\frac{1}{2}, & 0\end{matrix}\right.\right)x^{\beta-\gamma-\frac{1}{2}}...dy$$

operatorni qo'llaymiz.

U holda

$$\tau(x)=\frac{\Gamma(\beta)}{\Gamma(2\beta)}2^{\beta-2}\frac{d}{dx}x^{-\frac{\beta}{2}-1}\int_0^x y^{-\gamma}G_{44}^{40}\left(\frac{y}{x}\left|\begin{matrix}\frac{1-2\beta}{2}, & \frac{1-4\beta}{2}-\alpha+\gamma, & \frac{\beta-1}{2}, & \frac{2+\beta}{2}\\ \frac{1-2\beta}{2}-\alpha+\gamma, & \frac{1-4\beta}{2}+\gamma, & -\frac{1}{2}, & 0\end{matrix}\right.\right)\times$$

$$\times\left[\left(\bar{a}_1(y)+\bar{a}_2(y)\right)\bar{v}(y)+\left(\bar{b}_1(y)+\bar{b}_2(y)\right)\right](y)^{\beta-\gamma-\frac{1}{2}}dy \quad (14)$$

tenglikni hosil qilamiz. Qaralayotgan yechim esa (14) formula bilan yoziladi.

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