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PIPELINE STRENGTH ANALYSIS UNDER HYDRAULIC HAMMER

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Abstract

The process of pipe deformation under the influence of increased pressure during a hydraulic hammer caused by a flow interruption, the stress state of a pipe with an outer and inner diameter under load, and the differential equations of motion for elastic deformations were determined. A mathematical model of a pressure system resistant to hydraulic hammer and parameters for determining the minimum pipe wall thickness, taking into account the deformation of the pipe material, were developed.

Keywords:Hydraulic shock, flow interruption, pressure increase, deformation, radial and elastic stress, elastic deformation, pipe wall thickness.

Introduction

РАСЧЕТ ПРОЧНОСТИ ТРУБОПРОВОДА ПРИ ГИДРАВЛИЧЕСКОМ УДАРЕ

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Аннотация

Определены процесс деформации трубы под воздействием повышения давления при гидравлическом ударе, вызванном прерыванием потока, напряженное состояние трубы с наружным и внутренним диаметром под нагрузкой, дифференциальные уравнения движения для упругих деформаций. Разработаны математическая модель напорной системы, устойчивых к гидравлическим ударам, и параметры для определения минимальной толщины стенки трубы с учетом деформации материала трубы.

Ключевые слова: Гидроудар, повышение давления, деформация, радиальное и упругое напряжение, упругая деформация, толщина стенки трубы.

Introduction

Water hammer occurs in the pressure pipelines of a pumping station, causing damage to pressure pipes, check valves, the pumping station building, and pumping units [1-5]. During pump switching processes, that is, during start-up and shutdown periods, a pressure surge associated with water hammer occurs in the pressure pipelines. When the motor stops, the pump speed, water flow, and pressure decrease, and after a while, the flow reverses. In cases where the flow in the pipeline is interrupted, the pressure surge is even greater [1-5].

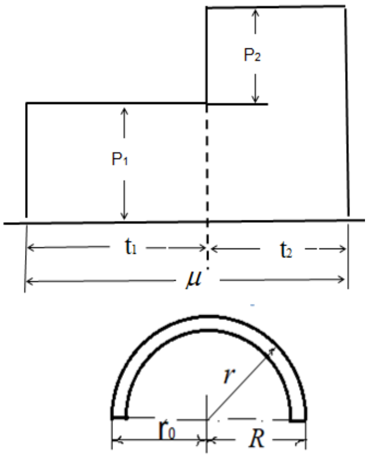

Fig.1.

During a hydraulic shock accompanied by a rupture of the flow continuity, two pressure surges are formed during the pressure increase phase [2]. Let us consider the process of pipe deformation caused by the action of instantaneous pressure increases along its internal boundary. Let us assume that at the moment of time t=0 the pressure instantly increases by the value p_1 and remains constant until the moment of time $t=t_1$, and at this moment a second pressure increase occurs, which remains constant until the moment of time $t=t_1+t_2$, when the pressure is instantly removed. It is necessary to determine the stress state of a pipe with an outer diameter R and an inner diameter r_0, under the action of the above load (Fig. 1) [1].

In [1], a problem was solved for plane deformation of a pipe, the material of which is assumed to be incompressible, under instantaneous application of a pressure p, which subsequently remains constant. The problem we posed differs from the problem presented in [1] in the short duration and stepwise nature of the applied load [2].

Research methodology

From the condition of incompressibility of the pipe material we have [2-5]:

$$\frac{\partial u}{\partial r}+\frac{u}{r}=0\,, \qquad (1)$$

Where u(r,t) - radial displacement of pipe points, r - distance of a point from the center of the pipe.

The solution to equation (1) will be

$$u(r,t)=\frac{c_1(t)}{r}\,. \qquad (2)$$

For small plastic deformations, the differential equation of motion has the form:

$$\frac{\partial \sigma_r}{\partial r}+\frac{\sigma_r-\sigma_\theta}{r}=\rho_0\frac{\partial^2 u}{\partial t^2}\,, \qquad (3)$$

Where σ_r and σ_θ - radial and hoop stresses, respectively, ρ_0 - density of the pipe material.

For elastic deformations

$$\sigma_r-\sigma_\theta=2A\left(\frac{\partial u}{\partial r}-\frac{u}{r}\right)=-4A\frac{c_1(t)}{r^2} \qquad (4)$$

and equation (3) is transformed to the form:

$$\frac{\partial \sigma_r}{\partial \sigma}=\rho_0\frac{c_1^{//}(t)}{r}+4A\frac{c_1(t)}{r^3}\,, \qquad (5)$$

Where A - Lame coefficient.

Integrating (5) under the boundary condition $\sigma_r(R,t)=0$ we obtain:

$$\sigma_r=\rho_0 c_1^{//}(t)\ln\frac{r}{R}-2A\frac{R^2-r^2}{R^2r^2}c_1(t)\,. \qquad (6)$$

On the inner surface of the pipe at $0\le t\le t1$, the stress $\sigma_r(r_0,t)=-p_1$, therefore

$$p_1=\rho_0 c_1^{//}(t)\ln\frac{R}{r_0}+2A\frac{R^2-r_0^2}{R^2r_0^2}c_1(t)\,. \qquad (7)$$

From expression (7) one can determine the function c(t) satisfying the initial conditions

$c_1(0)=c_1^{//}(0)=0$,

arising from natural initial conditions

$$u(r_0,0)=\frac{\partial}{\partial t}\left[u(r_0,0)\right]=0$$

The solution to equation (7) will be

$$c_1(t)=\frac{p_1 r_0^2 R^2}{2A\left(R^2-r_0^2\right)}\left(1-\cos\omega t\right), \qquad (8)$$

Where

$$\omega^2=\frac{2A\left(R^2-r_0^2\right)}{\rho_0 r_0^2 R^2\left(\ln R-\ln r_0\right)}. \qquad (9)$$

From equations (4), (8) and (9) we determine the stresses σ_r and σ_θ:

$$\sigma_r=-\frac{r_0^2R^2}{R^2-r_0^2}p_1\left\{\frac{1}{r^2}-\frac{1}{R^2}+\left[\frac{\left(R^2-r_0^2\right)\ln\frac{R}{r}}{R^2r_0^2\ln\frac{R}{r_0}}-\frac{R^2-r^2}{R^2r^2}\right]\cos\omega t\right\},$$

$$\sigma_\theta=-\frac{r_0^2R^2}{R^2-r_0^2}p_1\left\{\frac{1}{r^2}+\frac{1}{R^2}-\left[\frac{\left(R^2-r_0^2\right)\ln\frac{R}{r}}{R^2r_0^2\ln\frac{R}{r_0}}+\frac{R^2-r^2}{R^2r^2}\right]\cos\omega t\right\}. \qquad (10)$$

Let us assume that at the moment $t=t_1$ the pressure instantly increases by the value p_2 and becomes p_1+p_2. Then for $t_1<t<t_1+t_2$ on the inner surface of the pipe the radial stress will be $\sigma_r(r_0,t)=-p_1-p_2$ and according to equation (6), taking into account that $\sigma_r(R,t)=0$, we obtain:

$$p_1+p_2=\rho_0 c_2^{//}(t)\ln\frac{R}{r_0}+2A\frac{R^2-r_0^2}{R^2r_0^2}c_2(t). \qquad (11)$$

To obtain the initial conditions, we use the fact that at times t_1-0 and t_1+0, $c_1=c_2$ and $c_1^/=c_2^/$, that is:

$$c_2\left(t_1+0\right)=c_1\left(t_1-0\right)=\frac{r_0^2R^2p_1}{2A\left(R^2-r_0^2\right)}\left(1-\cos\omega t_1\right)$$

$$c_2^/\left(t_1+0\right)=c_1^/\left(t_1-0\right)=\frac{r_0^2R^2p_1\omega}{2A\left(R^2-r_0^2\right)}\sin\omega t_1 \qquad (12)$$

The solution of equation (11) taking into account conditions (12) will be represented in the form:

$$c_2(t)=\frac{r_0^2R^2}{2A\left(R^2-r_0^2\right)}\left[p_1\left(1-\cos\omega t\right)+p_2\left(1-\cos\omega(t-t_1)\right)\right]. \qquad (13)$$

Differentiating c2(t) twice and substituting the values $c_2(t)$ and $c_2^{//}(t)$ instead of $c_1(t)$ and $c_1^{//}(t)$ in equation (6) we obtain

$$\sigma_r = -\frac{\ln\frac{R}{r}}{\ln\frac{R}{r_0}}\left[p_1\cos\omega t + p_2\cos\omega(t-t_1)\right] - \qquad (14)$$
$$-\frac{R^2 - r^2}{R^2 - r_0^2}\frac{r_0^2}{r^2}\left[p_1(1-\cos\omega t) + p_2(1-\cos\omega(t-t_1))\right]$$

and taking into account equation (4)

$$\sigma_\theta = -\frac{\ln\frac{R}{r}}{\ln\frac{R}{r_0}}\left[p_1\cos\omega t + p_2\cos\omega(t-t_1)\right] + \qquad (15)$$
$$+\frac{R^2 + r^2}{R^2 - r_0^2}\frac{r_0^2}{r^2}\left[p_1(1-\cos\omega t) + p_2(1-\cos\omega(t-t_1))\right].$$

Elastic vibrations are possible under the condition

$$\left|\sigma_r - \sigma_\theta\right| = \frac{4Ac_2(t)}{r^2} < 2\sigma_0, \qquad (16)$$

where σ_0 - elastic limit.

From expression (16) it follows that the plastic state of the pipe material begins from the inner surface of the pipe ($r=r_0$) at the moment of time t^*, determined by the equality:

$$\sin\omega(t_0 + t^*) = \frac{p_1 + p_2 + \sigma_0\left(1 - \frac{r_0^2}{R^2}\right)}{\sqrt{p_1^2 + p_2^2 + 2p_1p_2\cos\omega t_1}}, \qquad (17)$$

Where

$$\sin\omega t_0 = \frac{p_1 + p_2\cos\omega t_1}{\sqrt{p_1^2 + p_2^2 + 2p_1p_2\cos\omega t_1}}. \qquad (18)$$

It is obvious that the transition of the pipe material to a plastic state is possible at $t^* \le t_1 + t_2$.

From expressions (17) and (18) it follows that time t^* depends on the duration of the first increase in pressure -t_1 and on the values p_1 and p_2.

It is of interest to determine the value of t1 at which t^* reaches a minimum. This makes it possible to determine the most unfavorable load. For this purpose, the right-hand sides of expressions (17) and (18) are expressed in terms of t_1.

We determined the values of p_1, p_2 and t_1 depending on m and ε.

$$p_1 = \gamma H_{\text{ст}} + \gamma\frac{a}{g}\vartheta_0 - 2\varepsilon\gamma\left(H_{\text{ст}} + h_b\right),$$

$$p_2 = 2\gamma\left(H_{\text{ст}} + h_b\right), \quad t_1 = \frac{(m+1)(1-\varepsilon)}{m+1-\varepsilon}\mu.$$

Taking these dependencies into account, equations (17) and (18) are transformed to the form:

$$\sin\omega\left(t_0+t^*\right)=\frac{\dfrac{H_{\text{ст}}}{2\left(H_{\text{ст}}+h_b\right)}+\dfrac{(m+1)^2-\dfrac{t_1}{\mu}}{2\left(m+1-\dfrac{t_1}{\mu}\right)}+\dfrac{\sigma_0}{2\gamma\left(H_{\text{ст}}+h_b\right)}\left(1-\dfrac{r_0^2}{R^2}\right)}{\sqrt{1+\left[\dfrac{H_{\text{ст}}}{2\left(H_{\text{ст}}+h_b\right)}+\dfrac{m^2-1+\dfrac{t_1}{\mu}}{2\left(m+1-\dfrac{t_1}{\mu}\right)}\right]^2+\left[\dfrac{H_{\text{ст}}}{H_{\text{ст}}+h_b}+\dfrac{m^2-1+\dfrac{t_1}{\mu}}{m+1-\dfrac{t_1}{\mu}}\right]\cos\omega t_1}},$$

$$\sin\omega t_0=\frac{\dfrac{H_{\text{ст}}}{2\left(H_{\text{ст}}+h_b\right)}+\dfrac{m^2-1+\dfrac{t_1}{\mu}}{2\left(m+1-\dfrac{t_1}{\mu}\right)}+\cos\omega t_1}{\sqrt{1+\left[\dfrac{H_{\text{ст}}}{2\left(H_{\text{ст}}+h_b\right)}+\dfrac{m^2-1+\dfrac{t_1}{\mu}}{2\left(m+1-\dfrac{t_1}{\mu}\right)}\right]^2+\left[\dfrac{H_{\text{ст}}}{H_{\text{ст}}+h_b}+\dfrac{m^2-1+\dfrac{t_1}{\mu}}{m+1-\dfrac{t_1}{\mu}}\right]\cos\omega t_1}}.$$

These expressions allow us to determine time

$$t^*=\frac{1}{\omega}\left(\arcsin\frac{\dfrac{H_{\text{ст}}}{2\left(H_{\text{ст}}+h_b\right)}+\dfrac{(m+1)^2-\dfrac{t_1}{\mu}}{2\left(m+1-\dfrac{t_1}{\mu}\right)}+\dfrac{\sigma_0}{2\gamma\left(H_{\text{ст}}+h_b\right)}\left(1-\dfrac{r_0^2}{R^2}\right)}{\sqrt{1+\left[\dfrac{H_{\text{ст}}}{2\left(H_{\text{ст}}+h_b\right)}+\dfrac{m^2-1+\dfrac{t_1}{\mu}}{2\left(m+1-\dfrac{t_1}{\mu}\right)}\right]^2+\left[\dfrac{H_{\text{ст}}}{H_{\text{ст}}+h_b}-\dfrac{m^2-1+\dfrac{t_1}{\mu}}{m+1-\dfrac{t_1}{\mu}}\right]\cos\omega t_1}}-\right.$$

$$\left.-\arcsin\frac{\dfrac{H_{\text{ст}}}{2\left(H_{\text{ст}}+h_b\right)}+\dfrac{m^2-1+\dfrac{t_1}{\mu}}{2\left(m+1-\dfrac{t_1}{\mu}\right)}+\cos\omega t_1}{\sqrt{1+\left[\dfrac{H_{\text{ст}}}{2\left(H_{\text{ст}}+h_b\right)}+\dfrac{m^2-1+\dfrac{t_1}{\mu}}{2\left(m+1-\dfrac{t_1}{\mu}\right)}\right]^2+\left[\dfrac{H_{\text{ст}}}{H_{\text{ст}}+h_b}-\dfrac{m^2-1+\dfrac{t_1}{\mu}}{m+1-\dfrac{t_1}{\mu}}\right]\cos\omega t_1}}\right).$$

Having plotted a graph of the dependence of t^* on t_1/μ within the range $0<t_1<1$, we determine the value $t^*_{\min}$.

The minimum thickness of the pipe wall can be determined from expression (17), assuming $t^*=\mu$, that is, considering that the plastic state of the pipe material is achieved at the end of the impact phase, when the pressure is instantly removed [2].

$$\sin\omega(t_0+\mu)=\frac{p_1+p_2+\sigma_0\left(1-\frac{r_0^2}{R_{\min}^2}\right)}{\sqrt{p_1^2+p_2^2+2p_1p_2\cos\omega t_1}}. \quad (19)$$

Taking into account the fact that under short-term loads the elastic limit of steel can be taken as $2\sigma_0$ [1,2,6-9], then the minimum wall thickness of the pipe determined by us using formula (19) will have a certain margin of safety.

Above we assumed that the pressure in the pipe increases instantly and remains constant in the time intervals t_1 and t_2. Taking into account the elasticity of the medium in which the pressure increases, it is obvious that it will not remain constant, but will depend on the displacement u, that is, p=p(u).

Let at the moment of time t=0 the volume of the pipe be equal to W_0 and the pressure instantly increases from 0 to p_0, and at moments t>0 due to the change in the volume of the pipe and the compressibility of the medium, the pressure p, the volume W [2].

Then we can write that

$$\frac{W-W_0}{W_0}=-\frac{p-p_0}{K},$$

Where K - bulk modulus of elasticity of the medium.

Considering that $W=\pi(r_0+u)^2l$, we obtain

$$\frac{2r_0u+u^2}{r_0^2}=-\frac{p-p_0}{K}.$$

Neglecting the term u^2 as a small value, we obtain

$$p=p_0-\frac{2uK}{r_0}. \quad (20)$$

Substituting the value p from (20) into equation (6) and taking into account that on the inner surface of the pipe the radial stress σ_r=-p, we obtain

$$p_0-\frac{2uK}{r_0}=\rho_0c^{//}(t)\ln\frac{R}{r_0}+2A\frac{R^2-r_0^2}{R^2r_0^2}c(t).$$

From this expression we determine the function c(t), satisfying the conditions c(0)=c/(0)=0

$$c(t)=\frac{p_0}{2A\frac{R^2-r_0^2}{R^2r_0^2}+\frac{2K}{r_0^2}}(1-\cos\Omega t),$$

Where

$$\Omega^2 = \frac{2A\dfrac{R^2 - r_0^2}{R^2 r_0^2} + \dfrac{2K}{r_0^2}}{\rho_0 \left(\ln R - \ln r_0\right)}.$$

Without defining the stresses σ_r and σ_θ, we will only indicate that elastic vibrations of the pipe are possible under condition (16), which in this case will be represented in the form:

$$\frac{p_0}{\dfrac{r^2}{r_0^2}\dfrac{R^2 - r_0^2}{R^2} + \dfrac{K}{A}}\left(1 - \cos\Omega t\right) < 2\sigma_0.$$

It follows that the pipe material passes into a plastic state from the inner surface of the pipe at the moment in time t^*, determined by the equality [2,10-13]:

$$\sin^2\frac{\Omega t^*}{2} = \frac{\sigma_0\left(1 - \dfrac{r_0^2}{R^2} + \dfrac{K}{A}\right)}{2p_0}. \qquad (21)$$

The solution to this problem under step loading can be performed similarly to the previous one [2].

Conclusion

Theoretical and experimental studies have shown that, during short-term power outages, due to disruption of flow continuity during a power outage, a pressure greater than the pump's capacity may develop in a pumping station pipeline. The duration of this pressure is shorter than the shock phase. A method for calculating the pipeline's strength under dynamic loading is presented. Pipe strength is determined by the minimum time required for the pipe material to transition to a plastic state under pressure. The minimum pipe thickness is determined from expression (21) [2].

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