ANALYZING PERTURBATION EFFECTS ON LOW SATELLITE ORBITAL ELEMENTS AT DIFFERENT TERMS OF EARTH POTENTIAL

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ABSTRACT

The space technology made it feasible to launch and operate man-made satellites for the purpose of studying the Earth's dimensions and shape from orbit or to examine them as targets from the surface of the world.

The Earth's gravitational field is not perfectly rigid; rather, its values alter slightly according to where you are on the planet. Its non-spherical shape and internal fluctuations in density are the cause.

Geoid is the name for the geometric body that represents the associated asymmetric gravity field in each of the directions. The largest deviation from the sphere is caused by the rotation of the Earth, which displaces the masses to the equator due to the centrifugal forces. The orbit of a spacecraft circling the Earth is influenced by these gravitational anisotropies, leading to slightly deformed Keplerian orbits. the variations of the orbital elements are highly Complicated The variations are governed by the oblateness variations with different harmonics factor (n = 2,4,10,30,50) which known as Zonal harmonics are defined by zeroth order (m = 0), where the dependence of the potential on longitude vanishes and the field is symmetrical about the polar axis.

The sensitivity of the satellites orbits is studied the of the gravity field models. the solution one set of six initial osculating orbital parameters is estimated together with the parameters in radial, along-track and out-of-plane directions.

The empirical orbit parameters estimated using different terms gravity coefficients (zonal) do not show significant differences in the Keplerian orbits components for short period analysis. The only noticeable differences are seen in the long period The magnitude of these effects is usually larger than the short-periodic variations from the zonal harmonics.

the orbital elements vary periodically and significantly over each orbit In closing we note that if the argument of perjee would not rotate by gravitational perturbations but would remain constant, then all orbital elements would grow unlimited. So, by and large it can be stated that the rotation of ω, primarily owing to Earth’s oblateness perturbations, has a stabilizing influence on orbits.

Keywords: Earth's Oblateness, Perturbations, Orbital elements, Zonal harmonic, gravitational potential.

Introduction

A perturbation is a deviation from some normal or expected motion. [1,2] the gravitational forces of other celestial bodies, in particular neighboring planets, or interactions with the space environment will perturb the Keplerian orbit around the central body.

In total there exist the following major disturbing forces:

- gravitational forces resulting from the non-spherical geometry and mass distribution of the central body
- gravitational forces of other celestial bodies (such as the Sun, Moon, planets)
- acceleration force resulting from the solar radiation pressure
- acceleration force resulting from the drag of the remaining atmosphere. [3]

best way to describe the Earth's external field, the numerical issues that come up when determining the field from artificial satellite motions that have been observed, and the problem of carefully comparing the field that is derived from these motions with that which is observed on the Earth surface. These problems may not have an impact on the main findings from observations of artificial satellites, such as our understanding of the Earth's flattening, but the increased accuracy in these observations demonstrates the need to take into account the finer points of theory and numerical calculations. Perhaps this is especially significant when studying the mechanics of the Earth's. [4]

When the orbit of the satellite is low Earth orbit (LEO), the perturbation due to oblateness of Earth and atmospheric drag plays very important role. Various analytic, semi-analytic and numerical techniques are adopted for solving perturbed equations of motion. King-Hele (1958) solved the equations of the motion of a satellite analytically by considering oblateness of Earth. (Stiefel and Scheifele (1972) They solved the equations of motion by applying KS transformations. The motion of satellite in the terrestrial upper atmosphere was studied by Sehnal (1980). The analytic solution of motion of satellite by considering combined effect of Earth's gravity and air drag was found by Delhaise (1991) using Lie transformations. The dynamics of satellite motion around the oblate Earth using rotating frame were developed by Yan and Kapila (2001). The Hamilton equations for the motion of satellite under the Earth's oblateness and atmospheric drag were derived and solved using canonical transformation by Khalil (2002). Hassan et. al. (2008) tried to find a solution of equations governing motion of artificial satellites under the effect of an oblateness of Earth by using KS variables. Al-Bermani et. al. (2012) investigated the effect of atmospheric drag and zonal harmonic J2 for the near-Earth orbit satellite namely Cosmos1484[5]. Ahmed K. Izzet et. investigated Moon and Sun perturbations Effects on The Orbital Elements of Earth Satellites Orbits [6]. The earth potential disturb on low-earth orbit satellites for both prograde and retrograde orbits determinned by ahmed k. izzet et [7] the study demonstrate that the inclination of the satellites affects how the earth's potential disturbance affects the orbital elements of low earth orbiting satellites, and that the effects of perturbations on prograde orbits are opposite to those caused on retrograde orbits .The Kepler problem treats the earth as if it is a spherical body of uniform density. In actuality, the earth's shape deviates from a sphere in terms of latitude (described by zonal harmonics), longitude (sectorial harmonics), and combinations of both latitude and longitude (tesseral harmonics) [8].

This paper aims to analyze how the gravitational potential, which can be represented by a set of spherical harmonic functions, affects earth satellites orbiting around a non-spherical Earth. It is preferable to first identify these impacts by looking for variations in six orbital elements and then examining changes in the satellites' geometrical (kinetic) positions.

1. Earth Gravity Field Model

The theoretical basis for establishing the geodetic datum is provided by the theory of the Earth's gravity field, or the theory of the Earth's form. The geoid is the name for the Earth's physical form. A normal ellipsoid represents the Earth's physical and mathematical shapes, while a reference ellipsoid represents the Earth's mathematical shape. The reference ellipsoid, also referred to as the normal ellipsoid, is a close approximation of the geoid. Therefore, in geodesy, the shape of the geoid serves as the primary point of reference for the shape of the Earth. The height of a particular location on Earth's surface can also be determined using the geoid as a reference surface.

Since the geoid is the level (equipotential) surface of the Earth's gravity field, processing of leveling data should take into account the properties of the theory of the Earth's gravity field. This section deals with the basic concepts of the theory of the Earth's gravity field [9]

The Earth can be regarded as a body constituted by infinite number of continuous point masses. The attraction that the Earth has exerted on the unit point mass F is the integral:

$$\vec{F} = -G \int_{Earth} \frac{1}{r^2} \frac{\vec{r}}{r} dm \qquad (1)$$

where dm is the differential mass element of the Earth and r represents the position vector between dm and the attracted mass, which is a variable of integration; the integral area is the total mass of the Earth. The direction of the gravitational attraction is toward the center of the Earth. Due to the rotation of the Earth, every point on the Earth experiences an inertial centrifugal force P The force of gravity of the Earth G is the resultant of the gravitational force acting upon a unit point mass and the centrifugal force of the Earth.

in geodesy, the concepts of gravity force and gravity acceleration are always used interchangeably. For any conservative gravity vector, there exists a so-called potential function such that the gradient of the function isthe gravity vector. The function of gravitational potential is a numeric function with respect to the variables of coordinate axes $X, Y, and\ Z$.

It can be proved that the gravitational potential of a body at an exterior point, given by [9]:

$$V = G \int_{M} \frac{dm}{r}, \qquad (2)$$

where dm denotes the differential mass element at the point (ξ, η, ζ); it is a variable of an integral (see fig 1)

$$r = \sqrt{(x-\xi)^2 + (y-\eta)^2 + (z-\zeta)^2} \qquad (3)$$

is the distance from dm to the attracted mass and the total mass of the integral area is M

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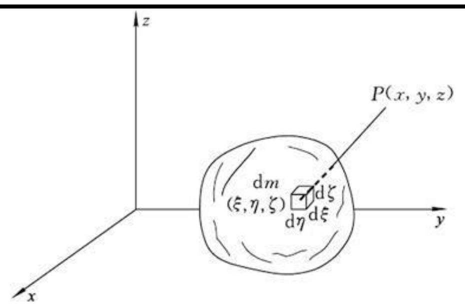


Fig (1) Gravitational potential of the body with mass M [9]

the harmonic function in the spherical coordinates, which can be represented as [10,11] :

$$V(\rho\,,\theta\,,\lambda\,) = \sum_{n=0}^{\infty} \frac{1}{\rho^{n+1}} \sum_{m=1}^{n} (a_{nm}\ \cos m\lambda + b_{nm}\ \sin m\lambda) p_{nm}(\cos\theta)\ , \qquad (4)$$

Equation is a series expansion, which indicates that the Earth's gravitational potential at an exterior point can be described by an infinite series. (ρ, θ, λ) are the spherical coordinates of the exterior point of the Earth and a_{nm} and b_{nm} are the coefficients of the Earth's gravity field, Coefficients a_{nm} and $b_{nm}$are related to the mass distribution and shape of the Earth [12]: which can be determined by the observed values. Therefore, the gravitational potential problem can be regarded as the problem of study of the coefficient of the gravitational potential. P_{nm} (cos θ) represents the associated Legendre polynomials (also known as the associated Legendre functions of the first kind), n is degree, and m is order [9].

In practice, the spherical harmonics expansion of the gravitational potential of the Earth is written as

$$V(\rho,\theta,\lambda) = \frac{GM}{\rho}\left[1 - \sum_{n=2}^{\infty}\left(\frac{a}{\rho}\right)^{n} J_n\, p_n(\cos\theta) + \sum_{n=2}^{\infty}\sum_{m=1}^{n}\left(\frac{a}{\rho}\right)^{n} (\bar{J}_{nm}\cos m\lambda + \bar{S}_{nm}\sin m\lambda)\bar{p}_{nm}(\cos\theta)\right], \qquad (5)$$

where a denotes the semi major axis of the Earth ellipsoid and $\bar{P}_{nm}(\cos\theta)$ is the fully normalized associated

Legendre polynomials, which differ from the associated Legendre polynomials only by a constant factor J_n, $\bar{J}_{nm}$, $\bar{S}_{nm}$ are the normalized coefficients Adopting this set of coefficients, the differences between the values of the polynomials changing along with n and m (degree and order) are slight, making it convenient to use. The coefficient that is independent of longitude in(5) is called the coefficient of zonal harmonics, while the coefficient that is longitude dependent is called the coefficient of tesseral harmonics

2. Perturbations

Deviations from an idealized, standard, or undisturbed motion are called perturbations. We generally think of the universe as being very predictable and regular. Due to perturbations from other bodies (like the Sun and Moon) and extra forces (such as a nonspherical central body and drag) that are not taken into account in Keplerian motion, the actual motion will differ from the ideal two-body route. Observe that we refer to the central body to demonstrate that these perturbation techniques are applicable to any central body. [6].

It is implied by the term perturbed motion that there is an unperturbed motion. The orbital motion of two spherically symmetric bodies, as represented by the equations of motion, is the definition of unperturbed motion in celestial mechanics. [1].

$$\ddot{r} = -k^2(m_0 + m)\frac{r}{r^3} \stackrel{\text{def}}{=} -\mu\frac{r}{r^3} \tag{6}$$

where μ = $k^2(m_0 + m)$. We follow the procedure which led to the conservation law of total angular momentum inthe N-body problem by multiplying eqn. (1) by the vector operator r× and obtain

$$r \times \ddot{r} = -\mu\frac{r \times r}{r^3} \stackrel{\text{def}}{=} 0 \quad , \tag{7}$$

the solution of which is known in terms of simple analytical functions. The constant μ is the product of the constantof gravitation and the sum of the masses of the two bodies considered. The numerical value of μ thus depends on theconcrete problem and on the system of units chosen. The perturbed motion of a celestial body is defined as the solution of an initial value problem of the following type .The perturbation term δf may be rather arbitrary. The resulting equations usually are referred to as the Gaussian perturbation equations

$$\ddot{r} = -\mu\frac{r}{r^3} + \delta f(t, r, \dot{r}) \,, \tag{8}$$

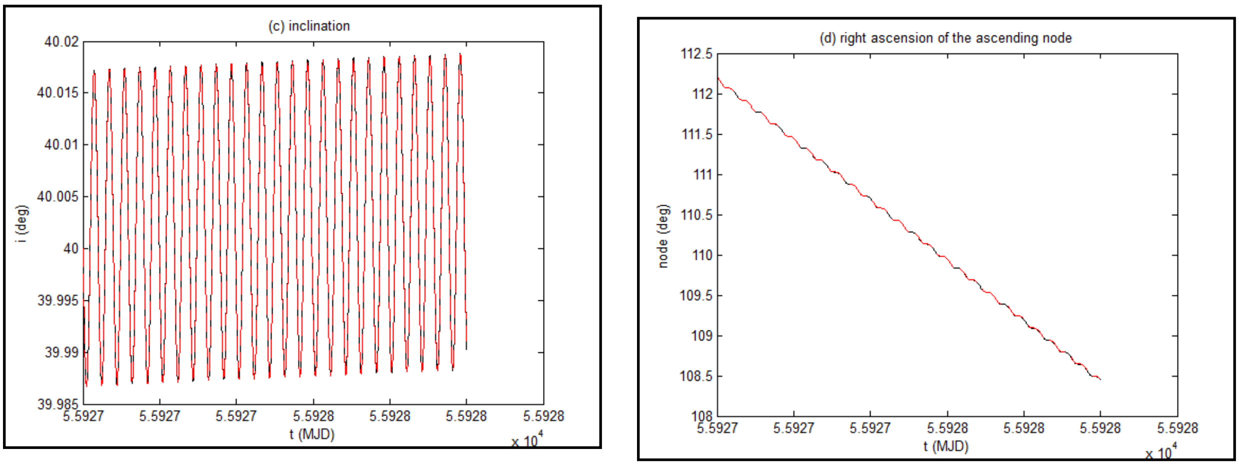
$$r(t_0) = r_0 \qquad and \qquad \dot{r}(t_0) = v_0 \tag{9}$$

For a system of point masses there is one such equation for each of the bodies, except for the central body to which the position vectors refer. The term $-\mu\frac{r}{r^3}$ in eqn. (8) is called the two-body term, δf the perturbation term. The terminology makes sense if the perturbation term is considerably smaller than the two-body term. The perturbation term δf may be rather arbitrary. The resulting equations usually are referred to as the Gaussian perturbation equations

$$|\delta f| \ll \left|-\mu\frac{r}{r^3}\right| \tag{10}$$

The differential equation system (8) is called the system of perturbation equations or simply the perturbation equations. Every method solving the initial value problem (8, 9) is a called a perturbation method.

For general perturbation methods it is mandatory not to use the original equations of motion in rectangular coordinates, but to derive differential equations for the osculating orbital elements or for functions thereof. This procedure promises to make the best possible use of the (analytically known) solution of the two-body problem (6), because the osculating elements are so-called first integrals of the two-body motion



Special perturbation methods may be applied directly to the initial value problem (8, 9) or to the transformed equations for the osculating elements [13].
The orbital elements are no longer constant in the perturbative case; instead, they change with time. Assume for the moment that we are aware of the orbital elements that, at a specific time (t), represent the Keplerian orbit, which, in general, describes the particle's motion.To solve the perturbed problem there exist various methods, which are usually divided in two main classes:

♦special perturbation methods, which integrate numerically a given problem with given initial conditions and therefore aim at obtaining specific solutions holding methods of numerical integration.

♦ general perturbation methods, aiming to provide approximate analytical solutions, no matter on the initial conditions. [13].

Moreover, as we will, the orbital perturbations can be classified in

□ secular : the orbital element affected by the perturbation varies linearly with time (or according to some power oftime);

□ long-period: the effects due to the perturbation repeat with a period which is at least 1 order of magnitude largerthan the orbital period;

□ short-period: the effects due to the perturbation repeat after few or less orbital periods[6].

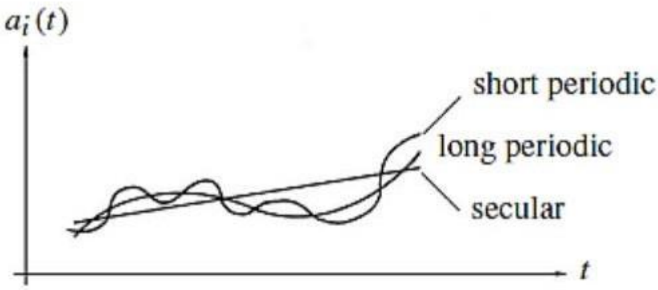
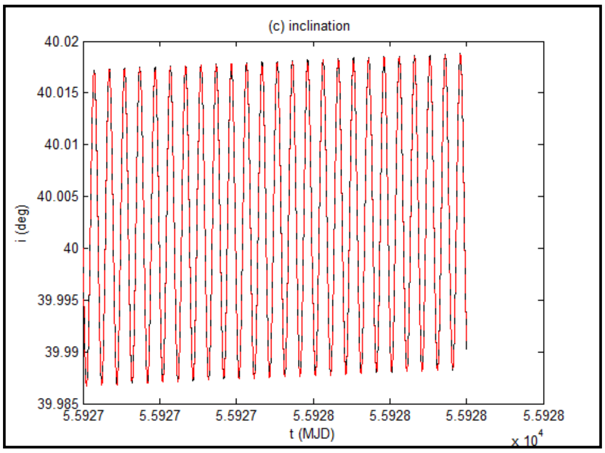


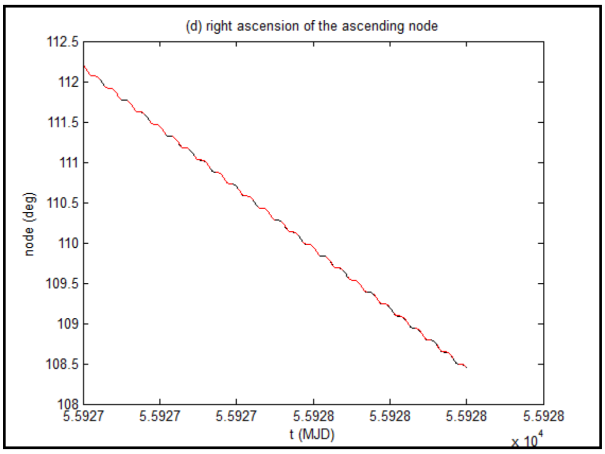
Figure (2) perturbations of the of the Keplerian element ai(t) [14] [15]

As the earth is not actually completely spherical, its gravitational potential is not constant for the same altitude. It depends on the region under consideration. The first step in modeling the perturbation is to obtain the analytical function of the potential energy created by a body of generic shape. Later, with the gravitational potential we can obtain the acceleration by making $\vec{a} = \overrightarrow{\partial U} / \overrightarrow{\partial r}$ and thus be able to model said force. Equation (11) shows the gravitational potential created by a body of generic shape:

$$U = \frac{\mu}{r}\left[1 - \sum_{n=2}^{\infty}\sum_{m=2}^{n} J_{nm}\left(\frac{R\oplus}{r}\right)^{n} \; P_{n,m}(\sin(\emptyset)).\cos\big(m(\lambda - \lambda_{n,m})\big)\right] \qquad (11)$$

Equation (11) is made up of the subtraction of two terms. The first term is $\frac{\mu}{r}$ and corresponds to the gravitationalpotential created by a sphere, that is, the spherical gravitational potential. The





other is the sum of the sphericalharmonics, which are corrections to the spherical model that allow characterizing the generic model [16]

The mathematical expression of this polynomial in equation (11) is given by the following [16]:

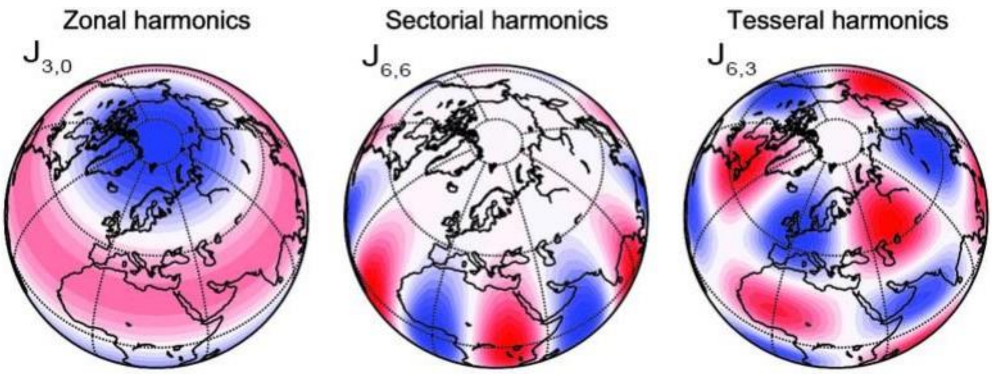
$$P_{n,m}(x) = \frac{-1^m}{2^n.n!}(1-x^2)^{m/2}\frac{d^{n+m}}{dx^{n+m}}(x^2-1)^n \qquad (12)$$

For m = 0, Legendre's associated polynomial takes the form of Legendre's polynomial:

$$P_n(x) = \frac{1}{2^n.n!}\frac{d^n}{dx^n}(x^2-1)^n \qquad (13)$$

Depending on the combination of n and m, we distinguish between three harmonics: zonal (m = 0), sectorial (n = m)

and tesseral (n ≠ m≠ 0). The zonal harmonics depend only on latitude. The sectorial harmonics depend both on latitude and longitude, though their sign is only dependent on longitude. The tesseral harmonics take values and signs that change regularly with respect to latitude and longitude, forming a chessboard-style pattern. Its graphical interpretation is illustrated in figure (2)[16].



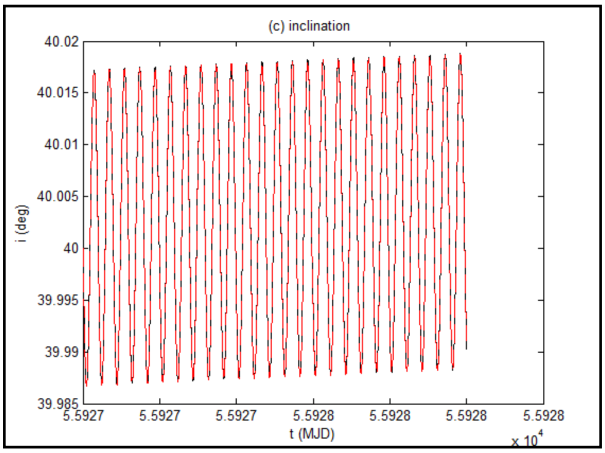
Figure(3)spherical harmonics (zonal, sectorial, and tesseral harmonics of the Earth) [17]

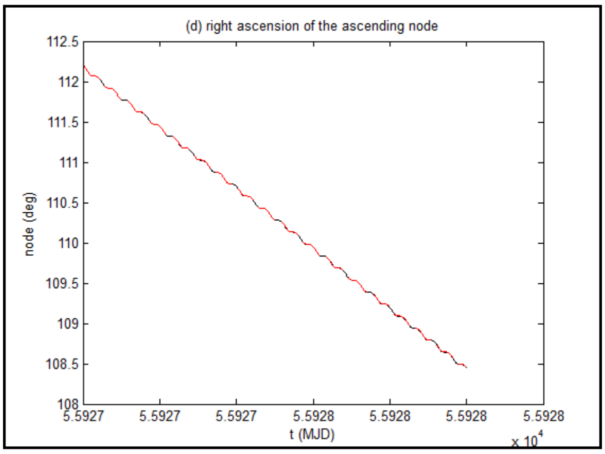
spherical harmonics, for they are periodic on the surface of a unit sphere. These spherical harmonics can be classifiedin three groups:

♦ Zonal Harmonics: They are defined by zeroth order (m = 0). This implies that the dependence of the potential on longitude (λ) vanishes and the field is symmetrical about the polar axis, appearing n+1 horizontal zones dividing Earth. J2 is, by far, the strongest perturbation due to Earth's shape. [18].

♦ Sectorial Harmonics: They occur when the degree is equal to the order of the harmonic (n = m). When this happens, the polynomials are zero at the poles and Earth is divided into 2n meridians sectors.[18].

♦Tesseral Harmonics: For cases in which n≠ m≠0, the tesseral harmonics attempt to model specific regions of Earth that depart from a perfect sphere [18].





3. Results and Discussion:

In this paper we try to analysis the effect of non-spherical earth on earth satellites by the gravitational potential which can be expressed in terms of a series of spherical harmonic functions. These spherical harmonics are used to describe the variation of the actual, pear-shaped earth from the spherically symmetric earth of the two-body problem. The indices n and m are used to differentiate between zonal harmonics (m = 0), sectorial harmonics (n = m, m ≠ 0), and tesseral harmonics (n ≠ m, m ≠ 0).

The specific perturbative effects of zonal will be discussed in the following for different values of degree.

however, it is first desirable to determine these effects by detecting the variation in six orbital elements and the by analyze the changes in the geometrical (kinetics) position of the satellites the results based on spherical harmonic degree coefficient's (2,4,10,30 and 50), the zonal harmonics describe how the actual shape of the earth deviates from the symmetrical Kepler earth in terms of latitude, the density of the earth at one particular line of latitude may be different (either higher or lower) than at another line of latitude therefor the physical effects which arise due to the zonal harmonics are very important when taking a ground track of satellites.

Using the program SATORB [13] the orbit of a test satellite is integrated over a relatively small-time interval of one day and over a long period of a month and six month in an attempt to fix the order of magnitude of the perturbations experienced by a typical satellite. Table (1) provides information on the test satellite's initial osculating orbital components.

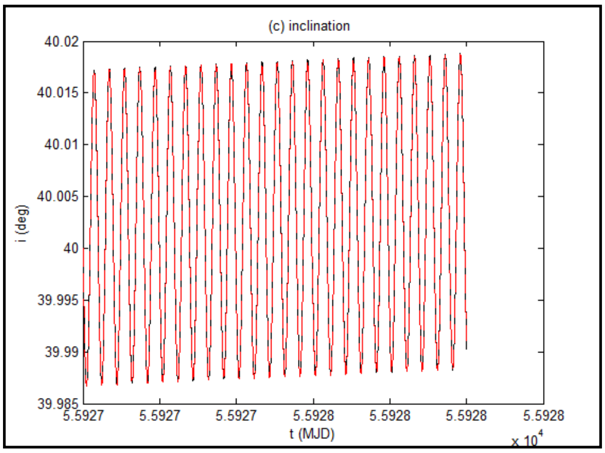
Table (1) osculating orbital elements

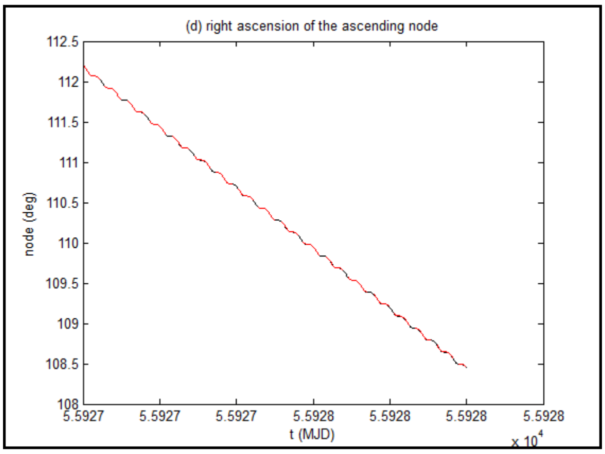
A (km)	e	I(deg)	Ω (deg)	Ω (deg)
7822	0.0006464	40	112.2	0

The initial epoch is (1 Jan 2012). The (unperturbed) revolution period is approximately (114.75) min. The mean oblateness perturbations are compared in Figures 4 through 17 to the mean perturbations obtained from an integration with the same beginning conditions (n = 2, m = 0), but with the Earth's gravitational potential up to different terms of degrees and set the order equal to 0. The osculating orbital elements extracted after the integration are given as a function of Modified Julian Date over the interval of the one day.

In general, as can be seen from the figures, that dominant effect on the (semi major axes, eccentricity, and inclination) elements is periodic, while we notice that secular variation beside the short periodic changes in the other remaining three elements. Secular perturbations in the longitude of ascending node, the argument of perigee, and the mean anomaly, these secular variations are the principal long-term effects of the non-spherical earth perturbation. The bulge at the equator produces a torque which tends to turn the satellite's orbit plane towards the equator; the satellite for the a spherical earth will cross the equator short of the crossing point for the unperturbed, two-body satellite, these phenomena, which is referred to as the regression of the node.

Secular perturbations in the longitude of ascending node, the argument of perigee, and the mean anomaly, these secular variations are the principal long-term effects of the non-spherical earth





perturbation. The bulge at the equator produces a torque which tends to turn the satellite's orbit plane towards the equator; the satellite for the a spherical earth will

cross the equator short of the crossing point for the unperturbed, two-body satellite, these phenomena, which is referred to as the regression of the node.

Figures (8 – 11) show that the gravitational potential term degree for period one month, changes exclusively the three elements: eccentricity, perigee argument, and mean anomaly. This is also accurate for intervals six months as shown infigures (12–15). This alteration remains unchanged when the term of degree is increased.

Secular changes dominate the ascending node's right ascent and the perigee argument. The findings demonstrated that the effects of perturbations on orbital elements are consistent with those caused by varying gravitational perturbation coefficient values, which motivates us to investigate turbulence over longer intervals.

The set of geocentric coordinates in the inertial system and the perturbing acceleration (in radial, along-track, and out-of-plane directions) are the variations with respect to the initial osculating Keplerian orbit can be measured in meters in the three directions shown in figure (16, 17). A secular variation coexists with the short periodic changes in along track direction, with the periodic effect having the most influence on the radial components. As we observe periodic expansion in the out-of-plane direction due to the influence of zonal harmonic coefficients.

4. Conclusion:

The results lead us to the conclusion that increasing the value of n for periods is directly proportional to time, Earth potential disturbance affects the orbital elements of LEO Satellites, the main effects on the orbit are the secular motions of the node and perigee. Periodic variations occur in all elements. The gravitational potential term degree for a one-month period is obtained from the data, and it just influences the eccentricity, perigee argument, and mean anomaly. This is also true for six-month durations.

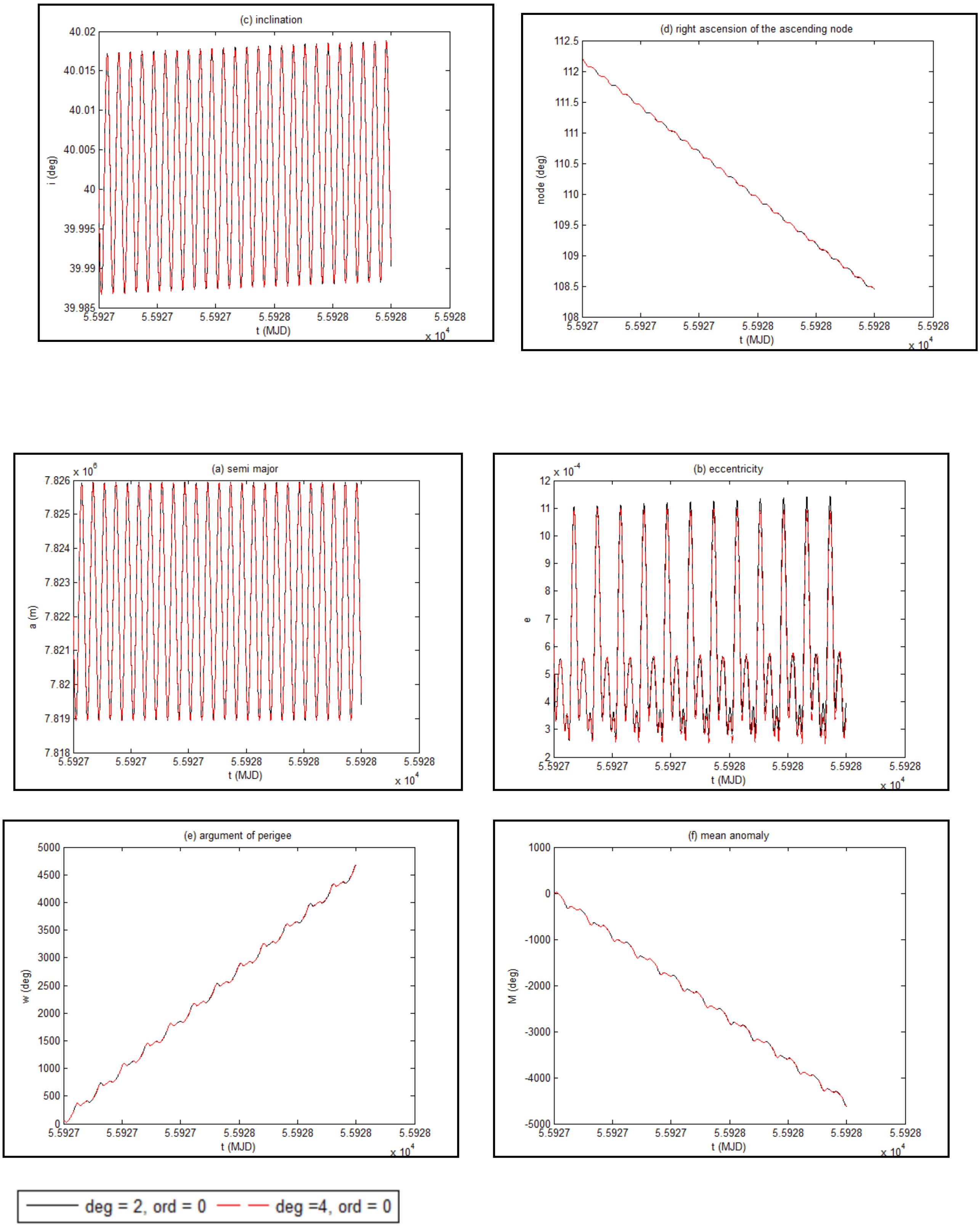
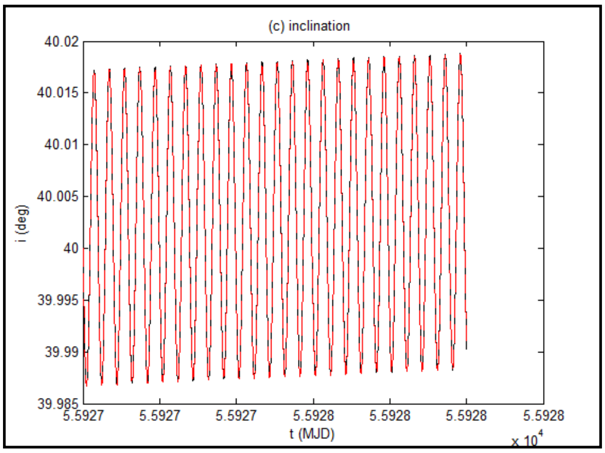
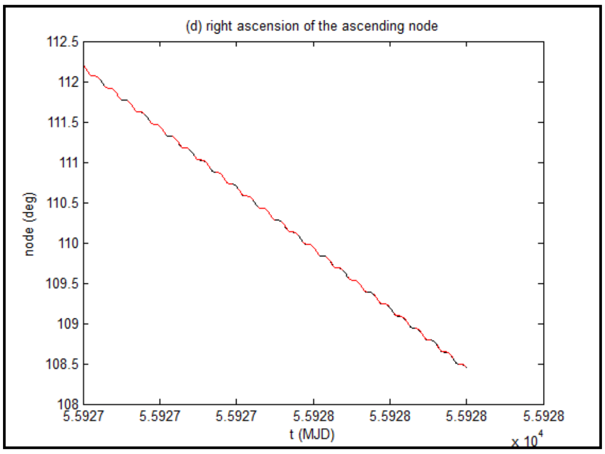
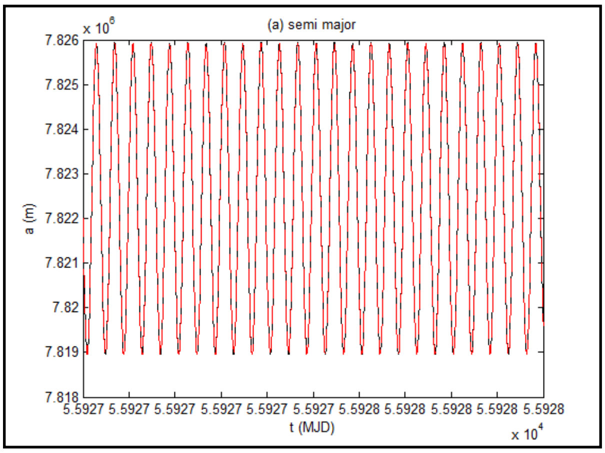
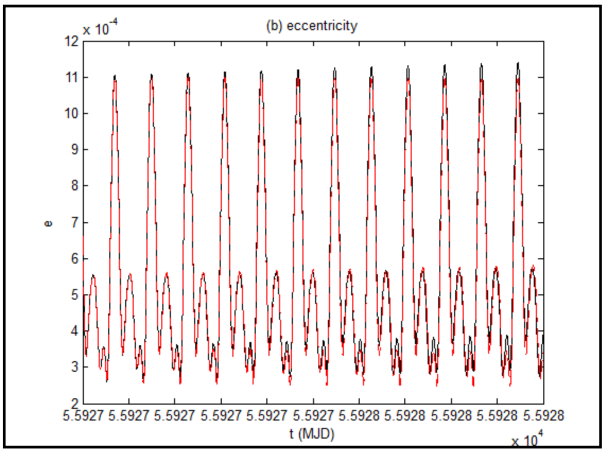


Figure (4) Variation in orbital element of a satellite due to the oblate Earth Zonal harmonic degree 4 (one day).


(c) inclination
40.02
40.015
40.01
40.005
40
39.995
39.99
39.985
i (deg)
5.5927
5.5927
5.5927
5.5928
5.5928
5.5928
5.5928
t (MJD)
x 10^4


(d) right ascension of the ascending node
112.5
112
111.5
111
110.5
110
109.5
109
108.5
108
node (deg)
5.5927
5.5927
5.5927
5.5928
5.5928
5.5928
5.5928
t (MJD)
x 10^4


(a) semi major
x 10^6
7.826
7.825
7.824
7.823
7.822
7.821
7.82
7.819
7.818
a (m)
5.5927 5.5927 5.5927 5.5927 5.5927 5.5927 5.5928 5.5928 5.5928 5.5928 5.5928
t (MJD)
x 10^4


(b) eccentricity
x 10^-4
12
11
10
9
8
7
6
5
4
3
2
e
5.5927 5.5927 5.5927 5.5927 5.5927 5.5927 5.5928 5.5928 5.5928 5.5928 5.5928
t (MJD)
x 10^4

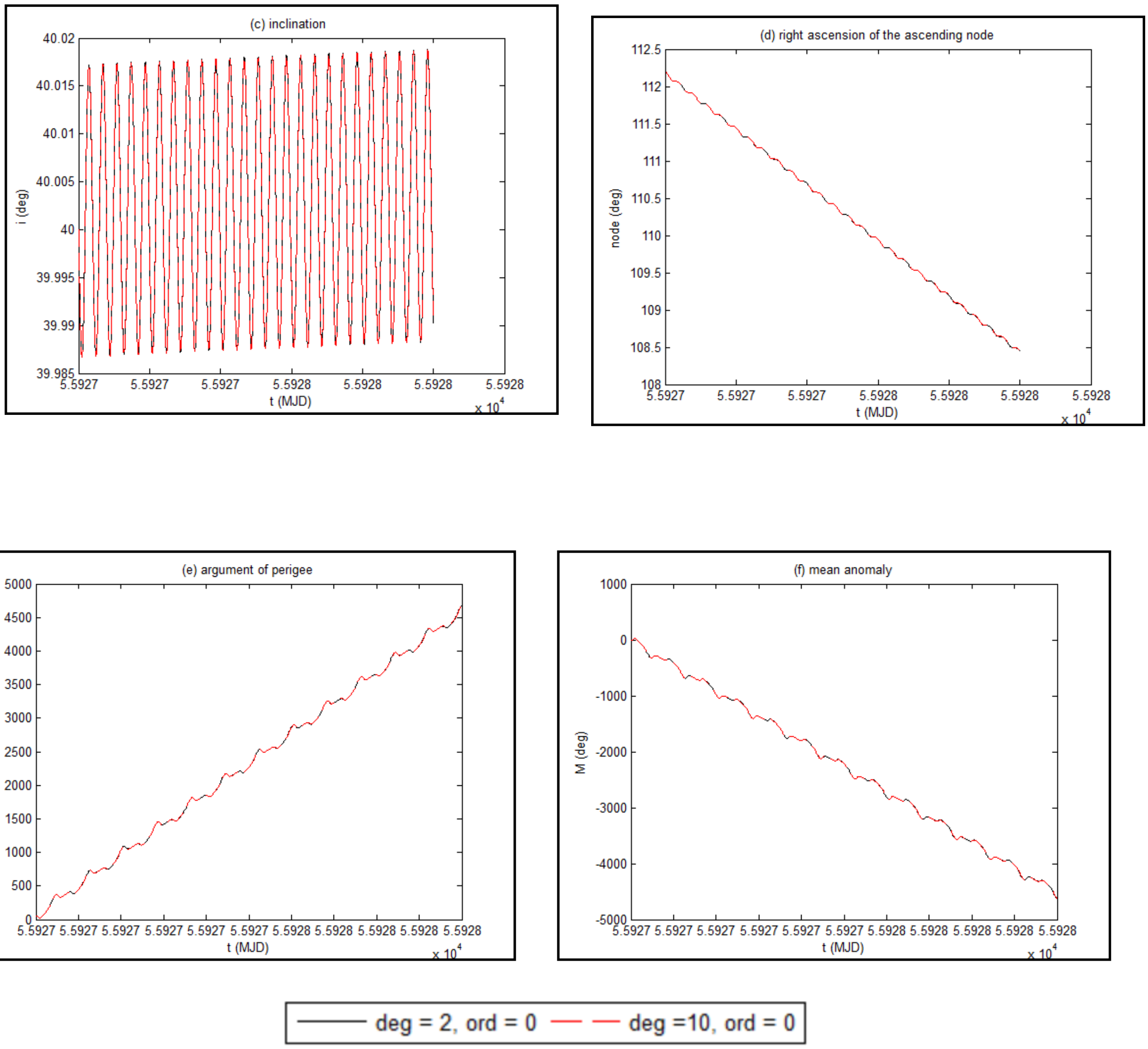
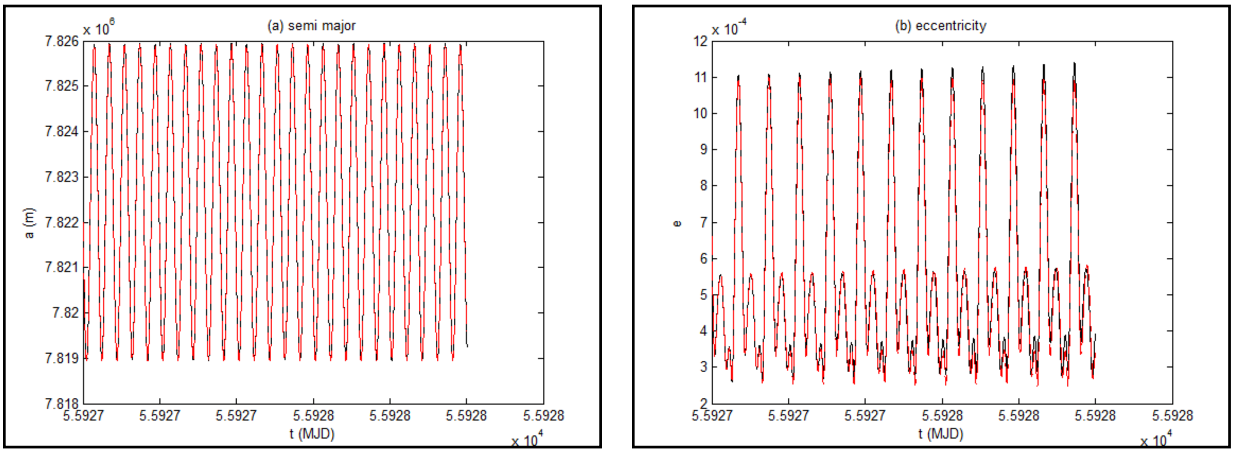


Figure (5) Variation in orbital element of a satellite due to the oblate Earth Zonal harmonic degree 10 (one day).



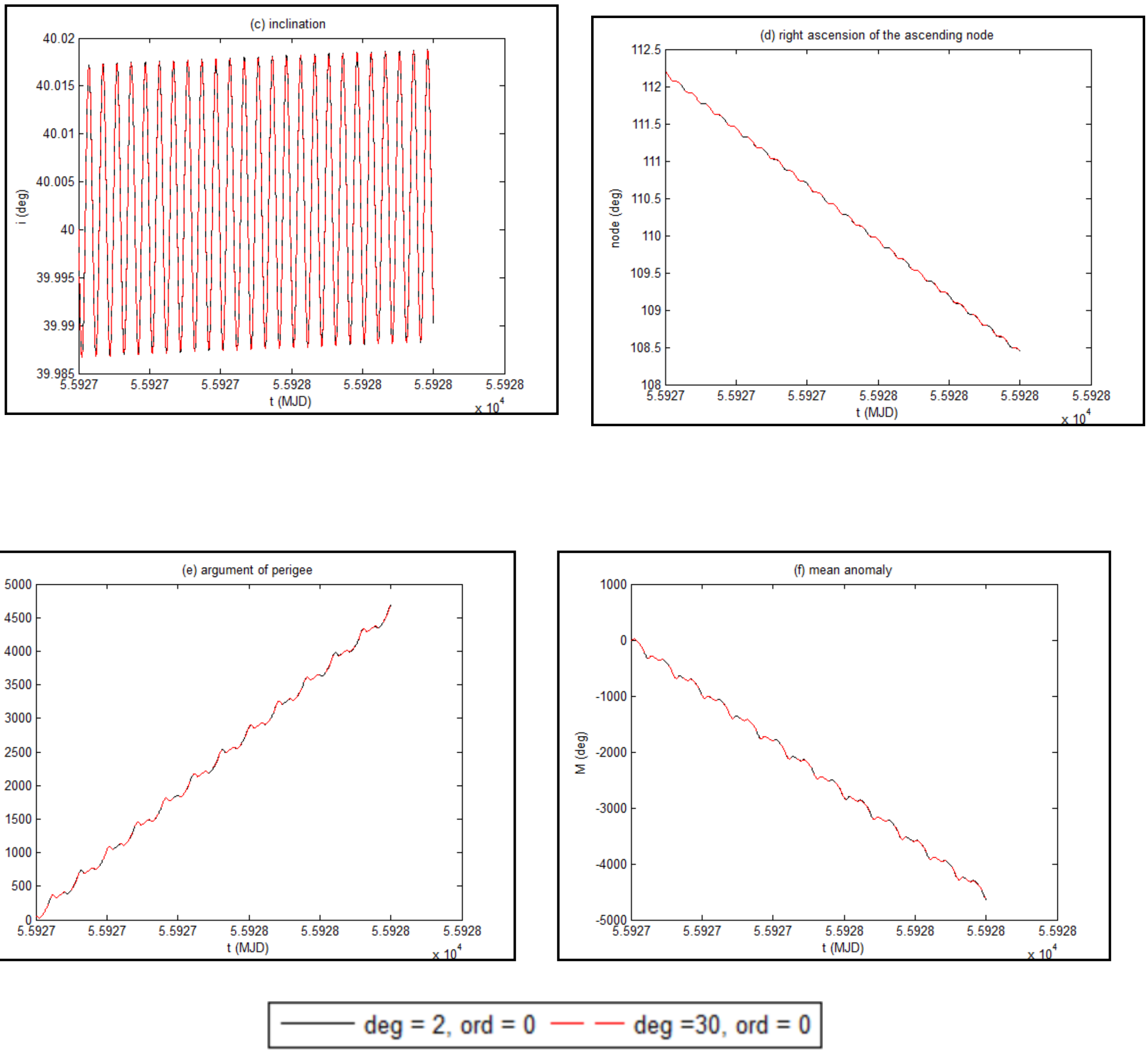
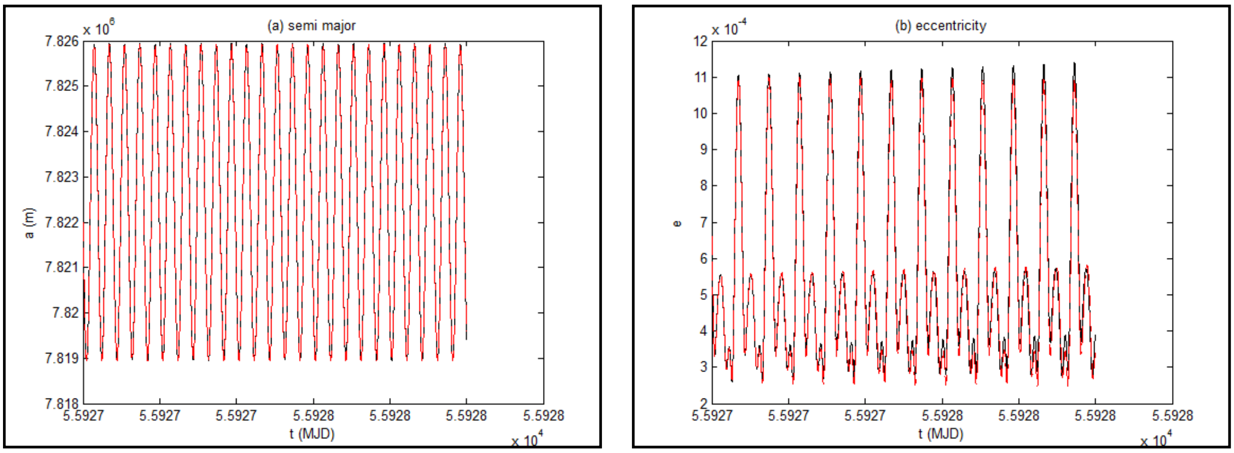


Figure (6) Variation in orbital element of a satellite due to the oblate Earth Zonal harmonic degree 30 (one day).



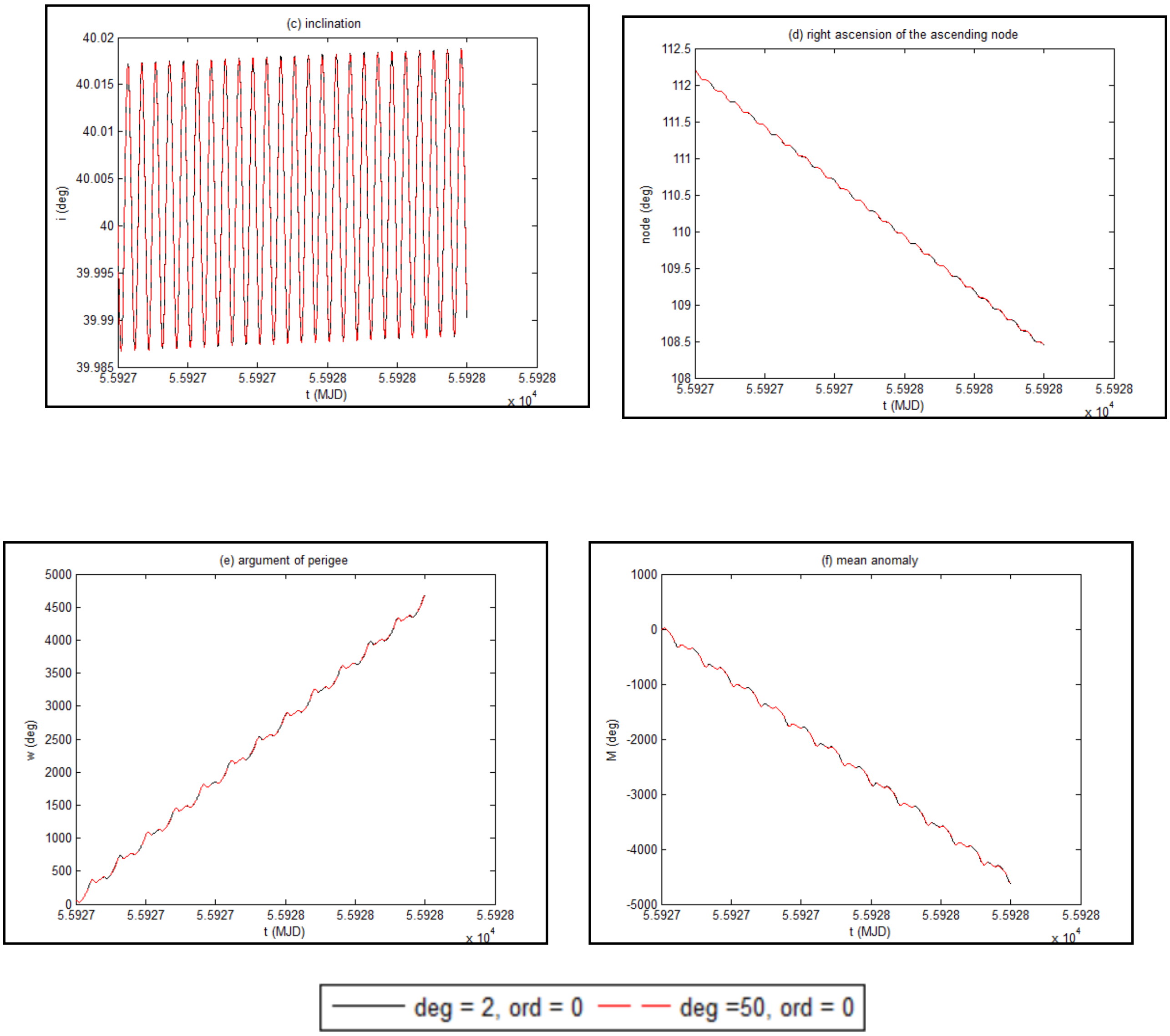
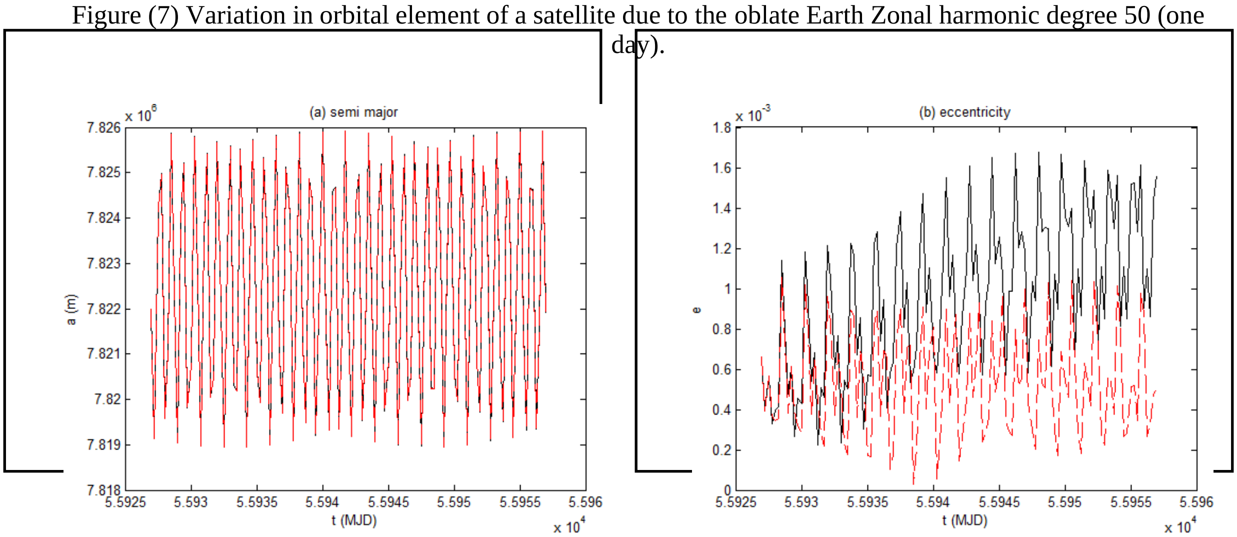


Figure (7) Variation in orbital element of a satellite due to the oblate Earth Zonal harmonic degree 50 (one day).



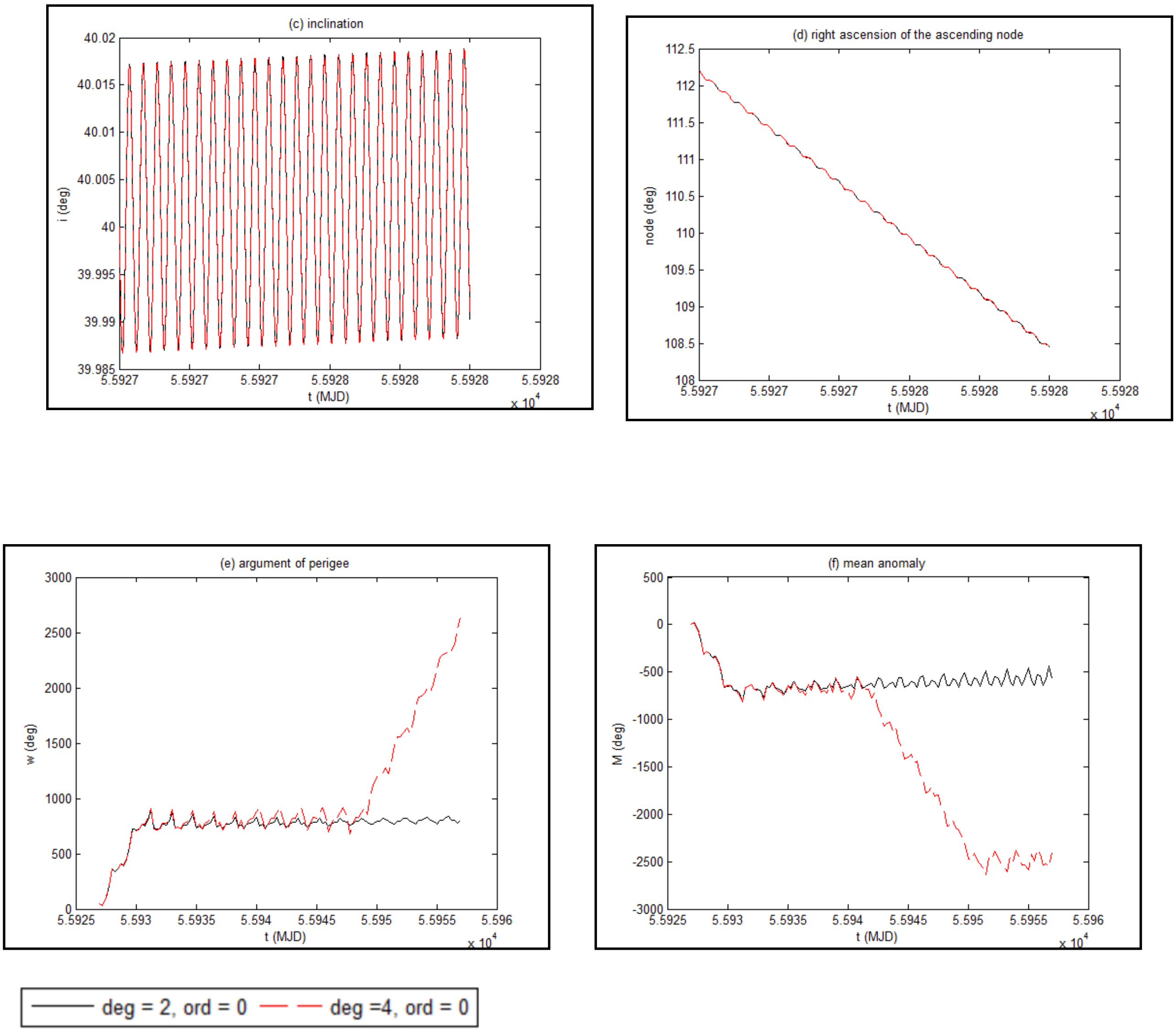
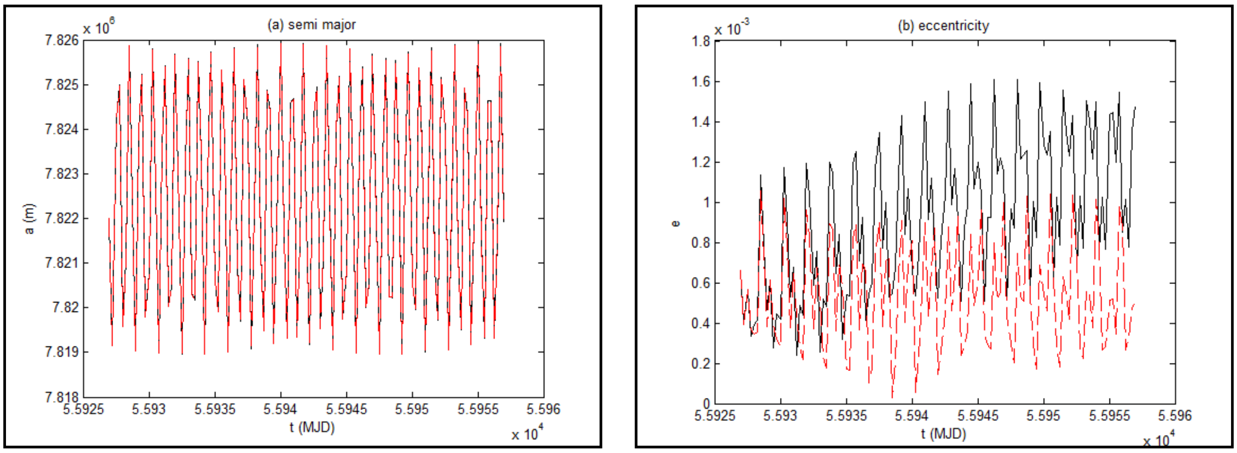


Figure (8) Variation in orbital element of a satellite due to the oblate Earth Zonal harmonic degree 4 (one month).



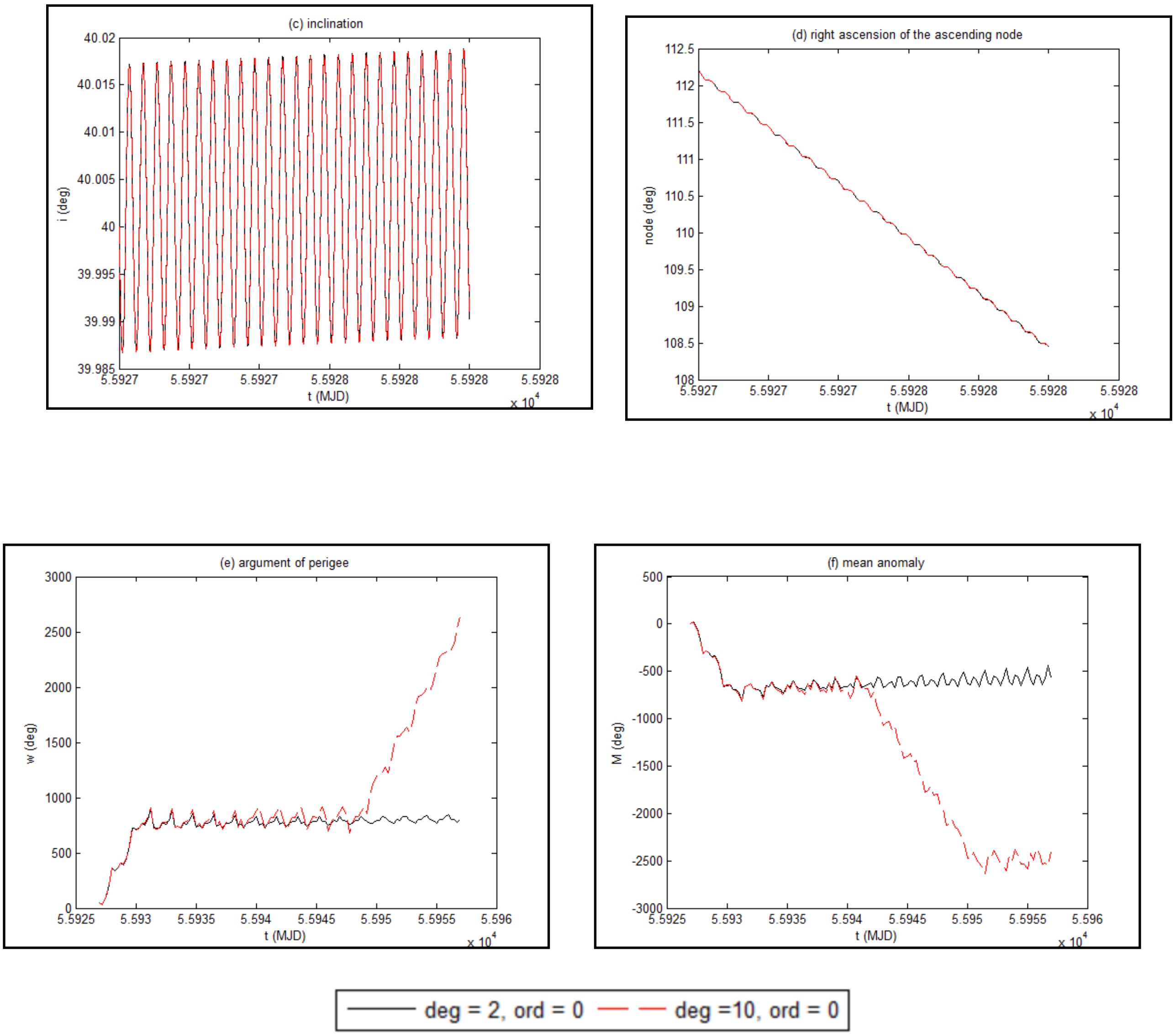
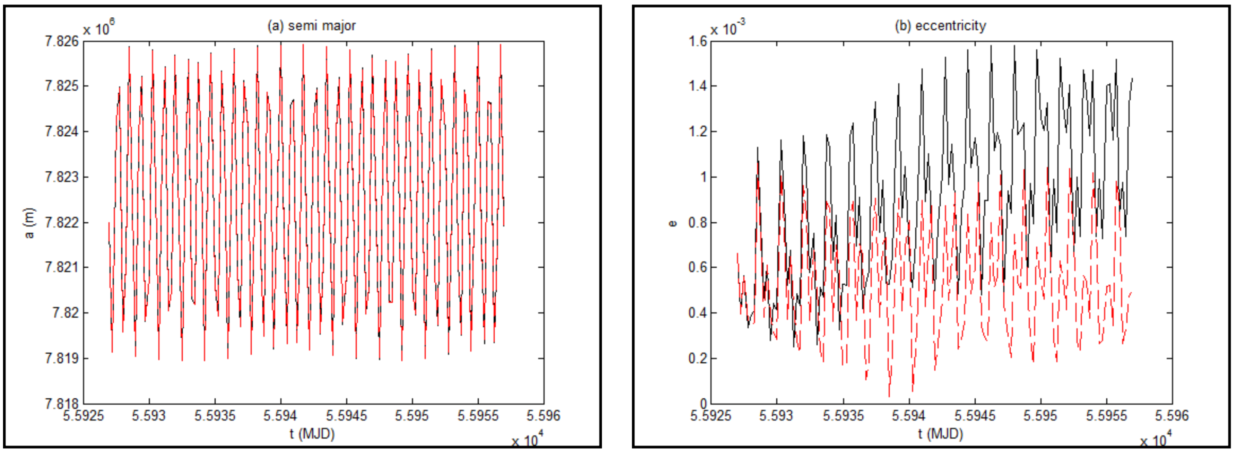
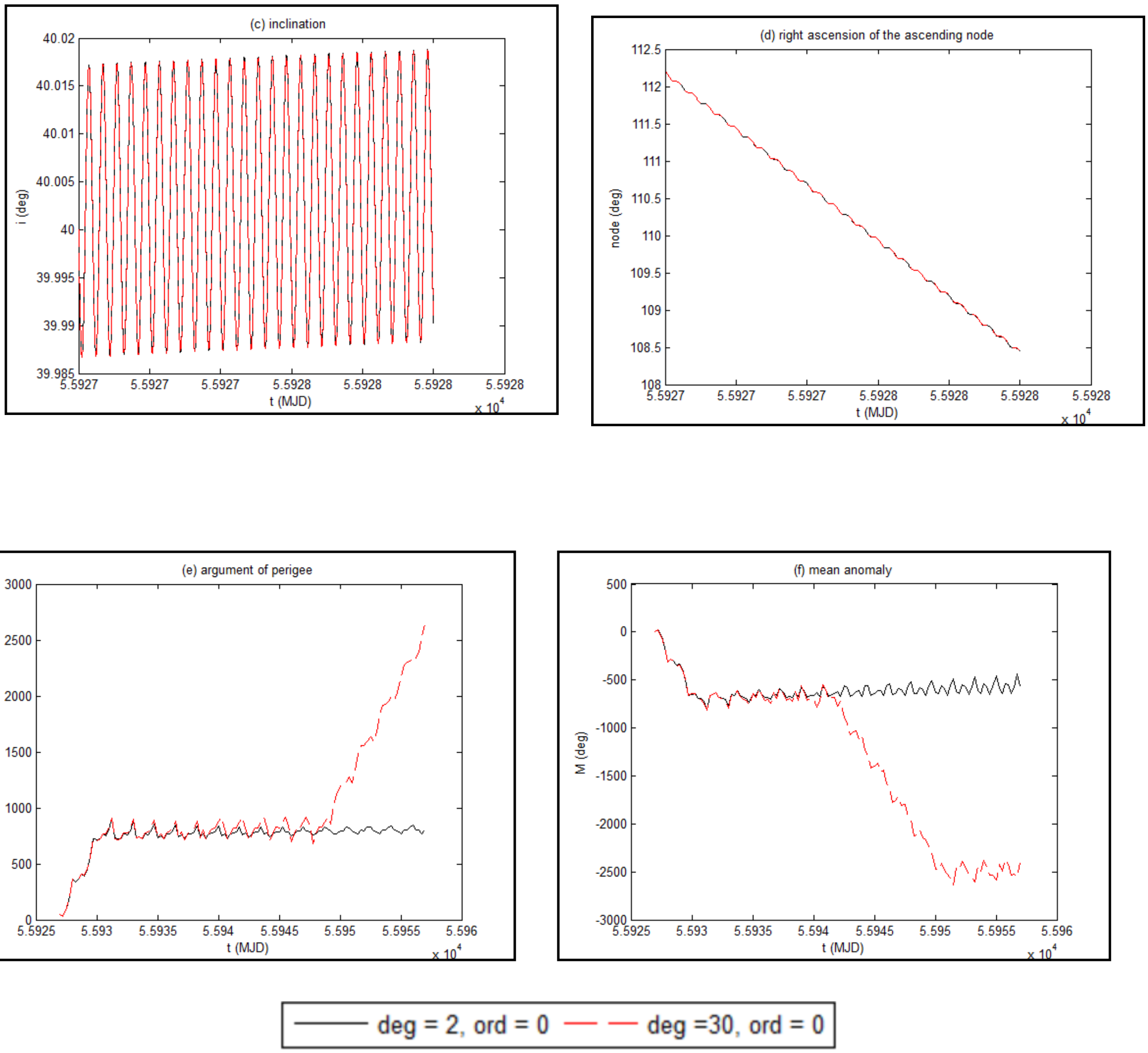
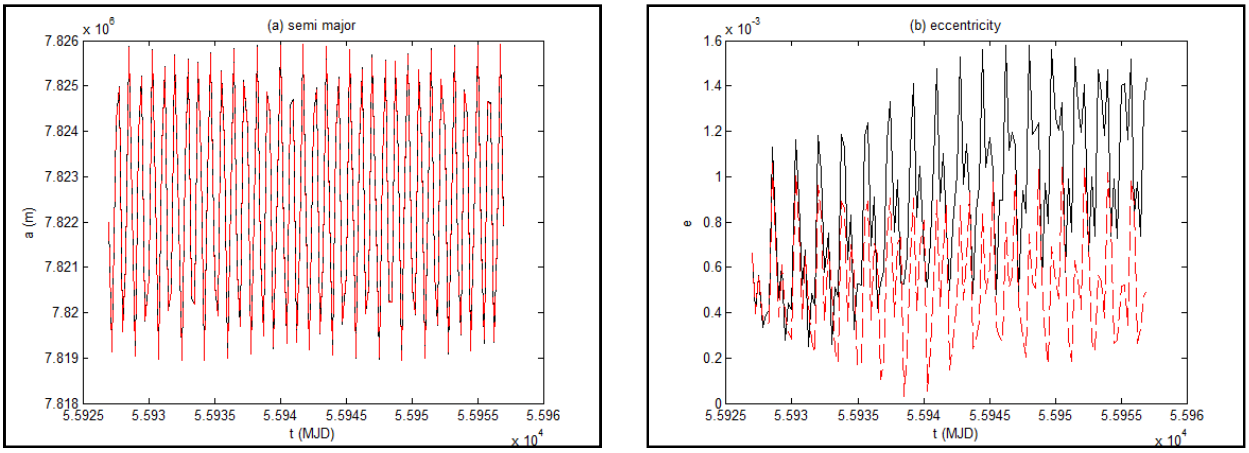


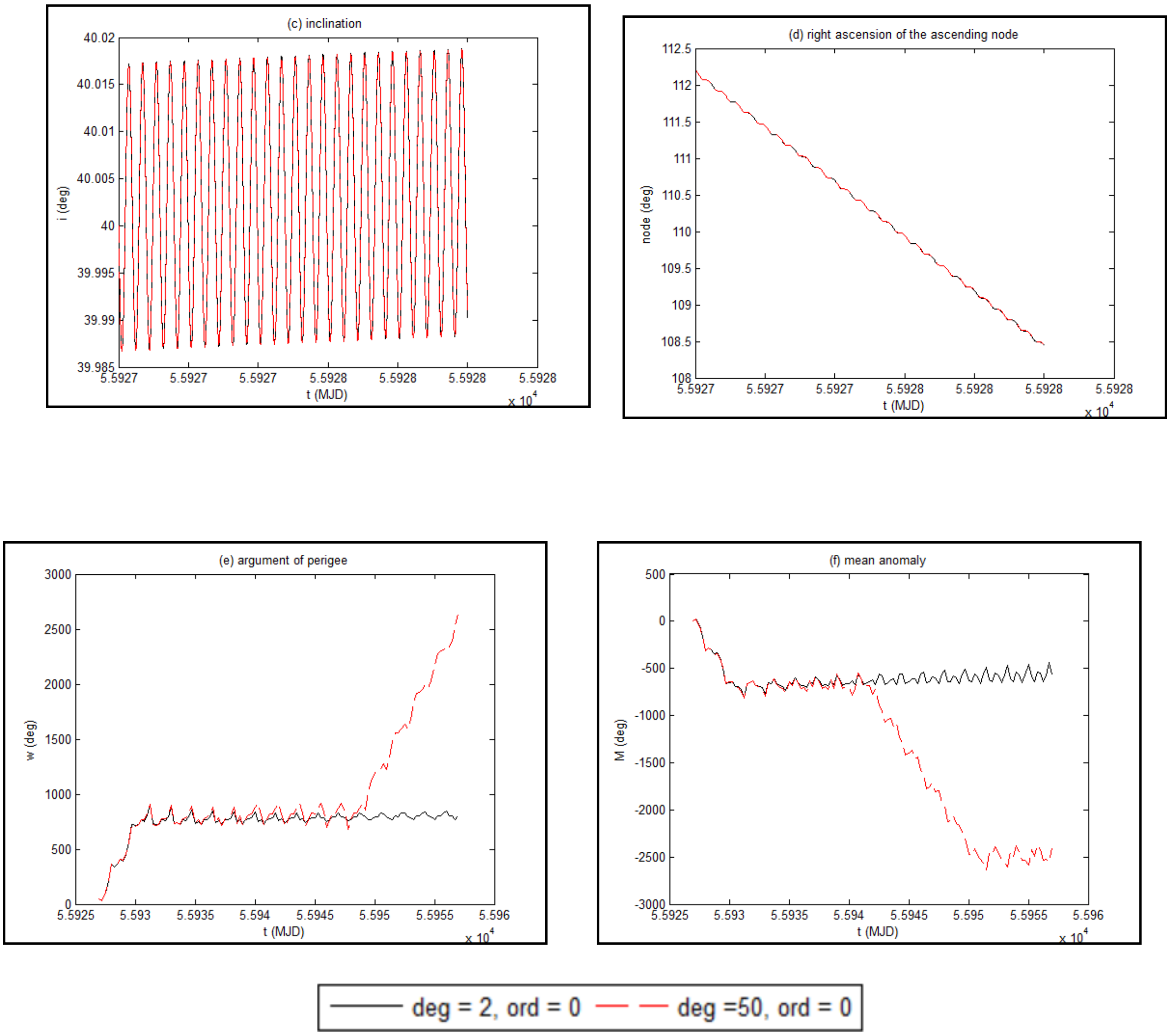
Figure (9) Variation in orbital element of a satellite due to the oblate Earth Zonal harmonic degree 10 (one month).



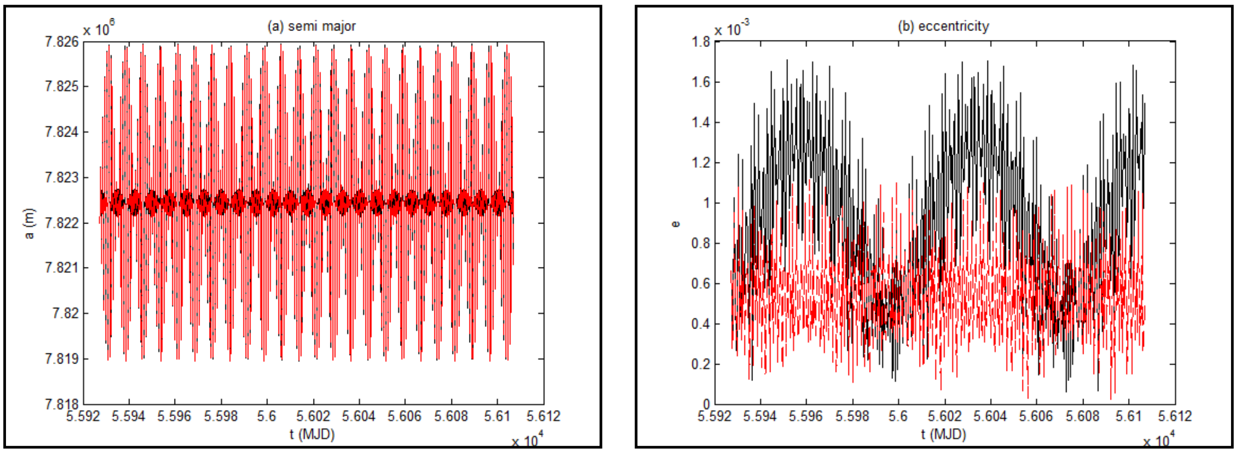


Figure(10)Variation in orbital element of a satellite due to the oblate Earth Zonal harmonic degree 30 (one month).





Figure(11) Variation in orbital element of a satellite due to the oblate Earth Zonal harmonic degree 50 (one month).



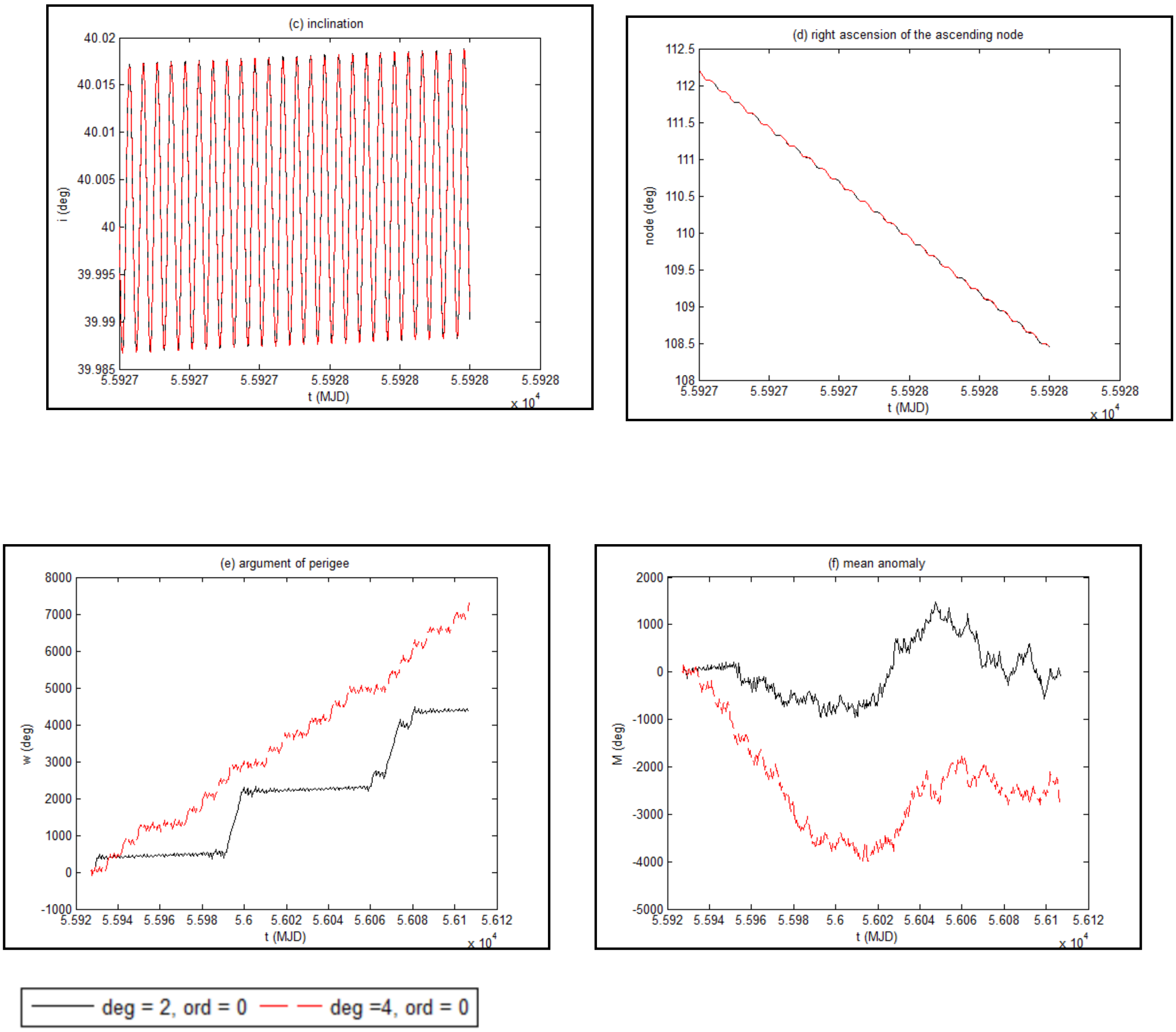
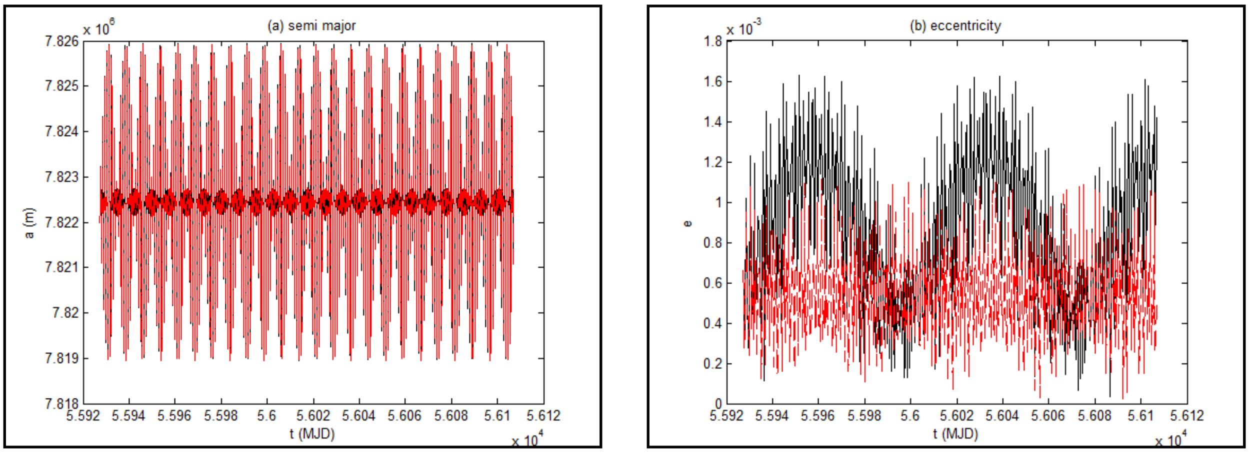


Figure (12) Variation in orbital element of a satellite due to the oblate Earth Zonal harmonic degree 4 (six month).



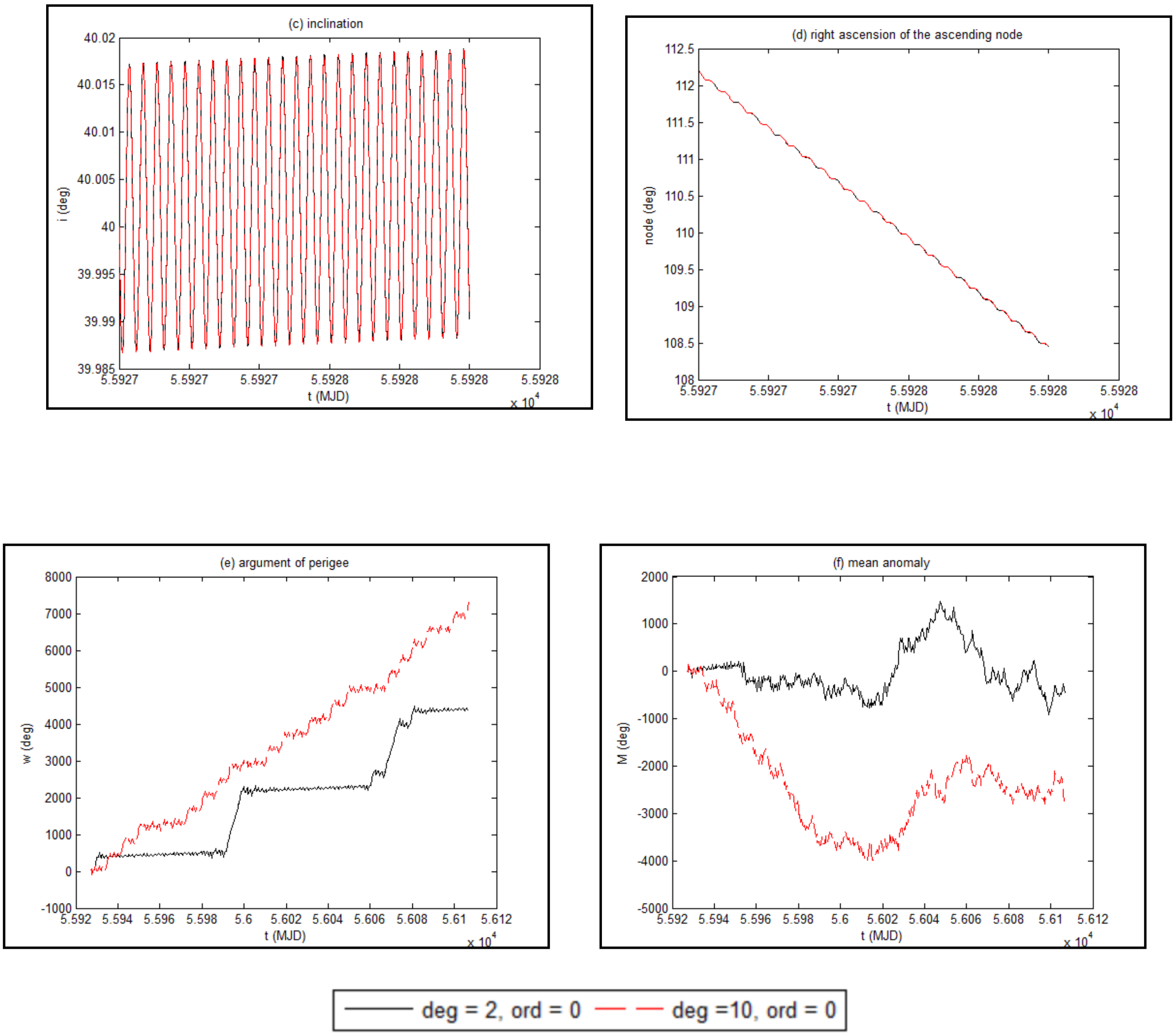
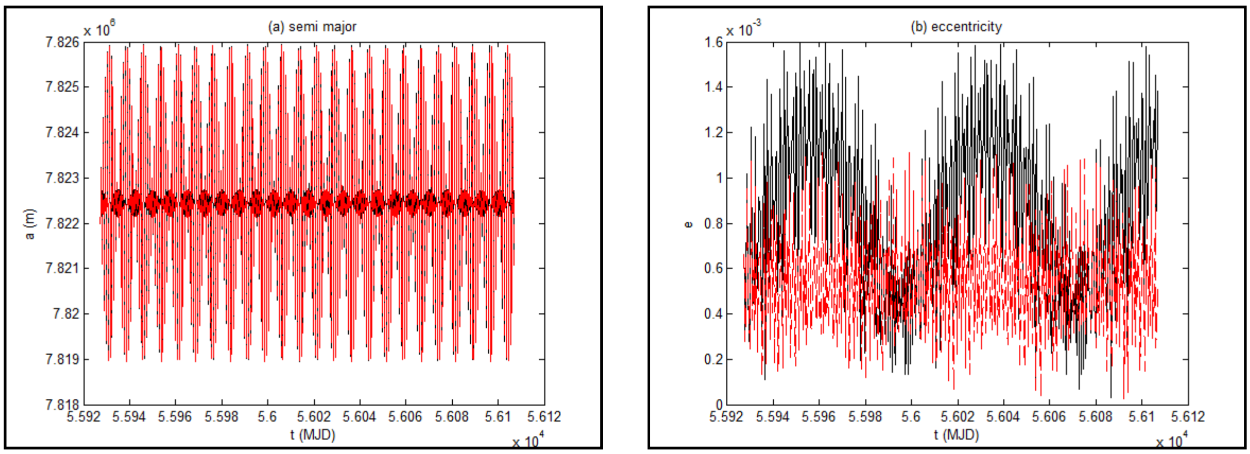


Figure (13) Variation in orbital element of a satellite due to the oblate Earth Zonal harmonic degree 10 (six month).



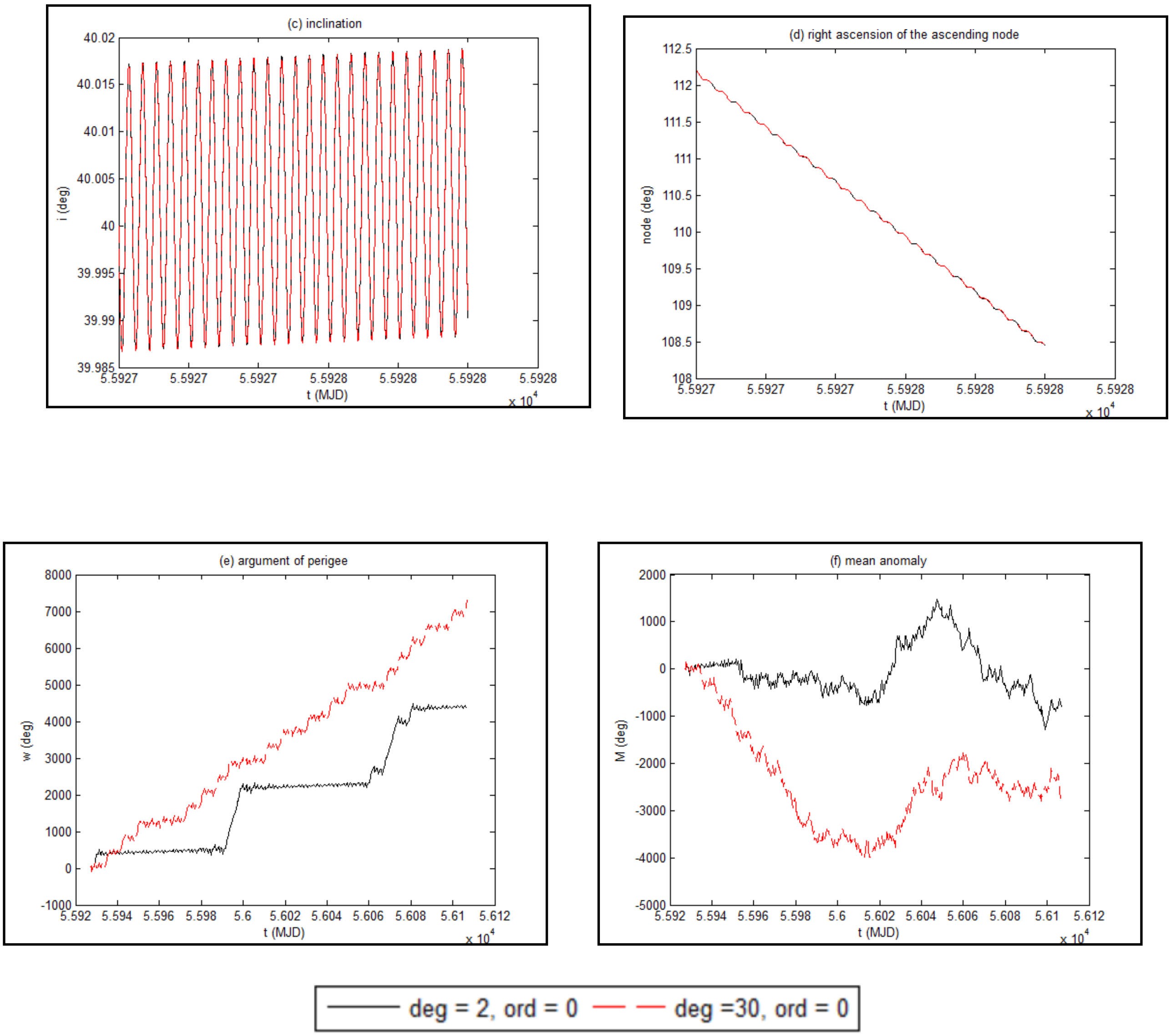
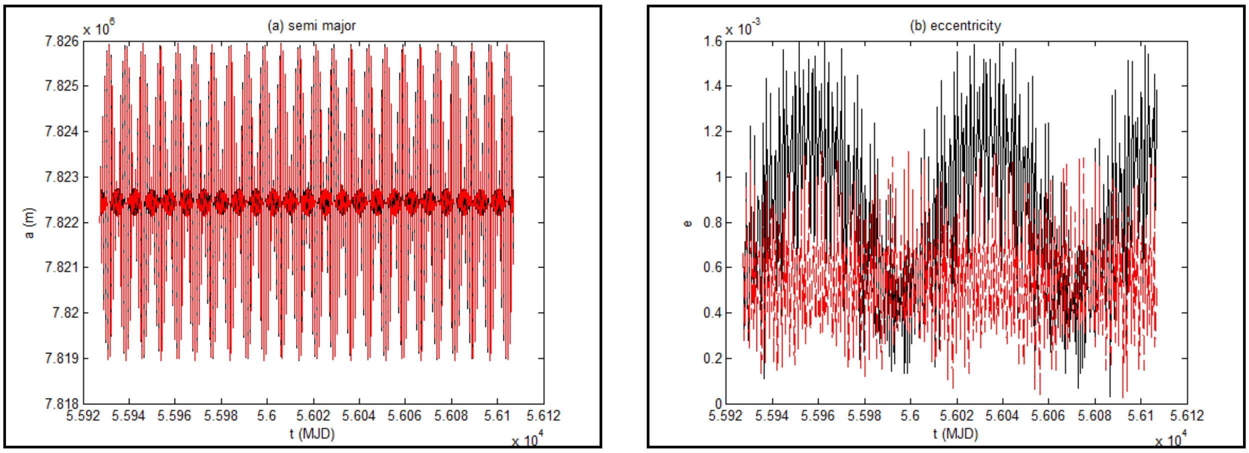


Figure (14) Variation in orbital element of a satellite due to the oblate Earth Zonal harmonic degree 30 (six month).



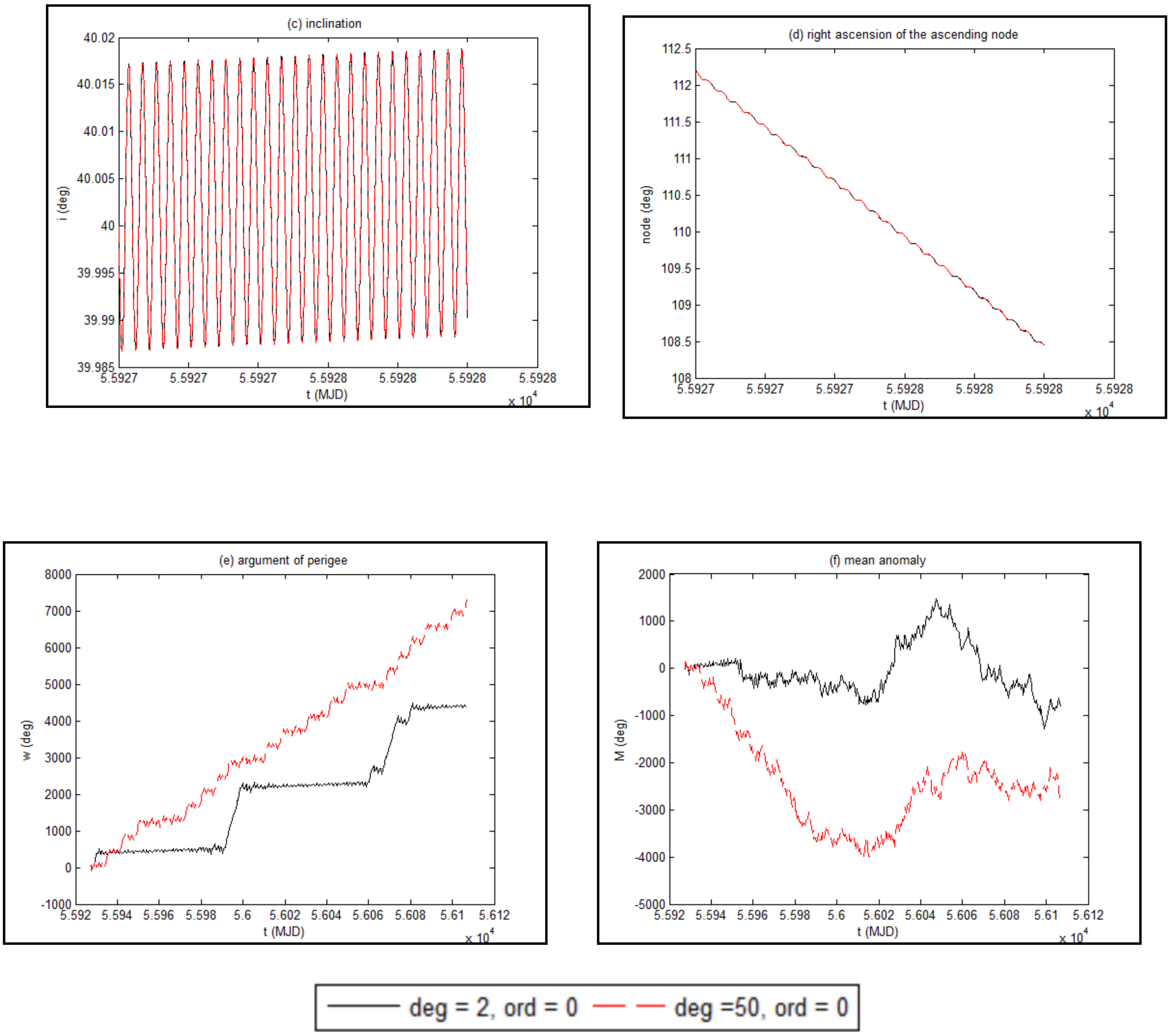
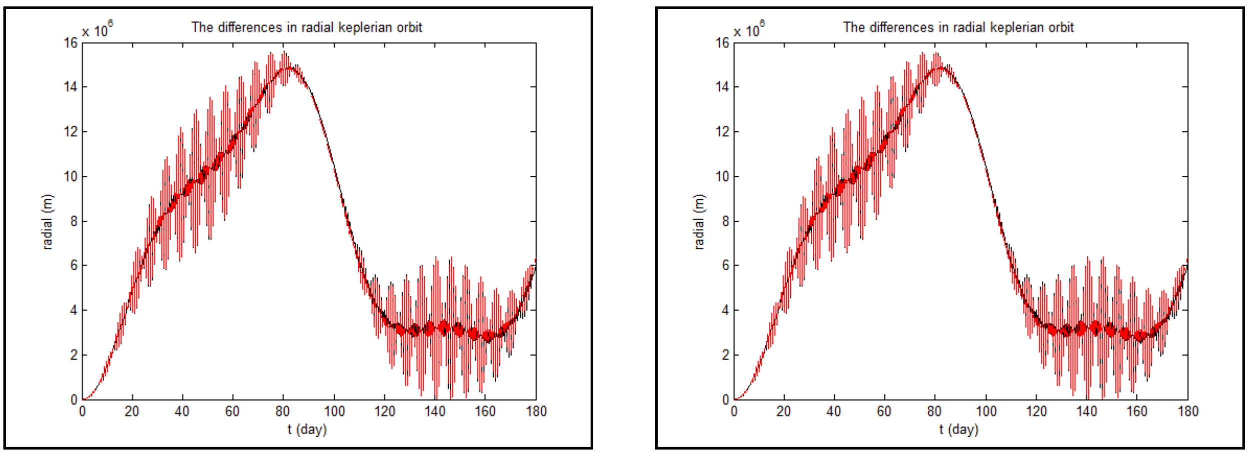
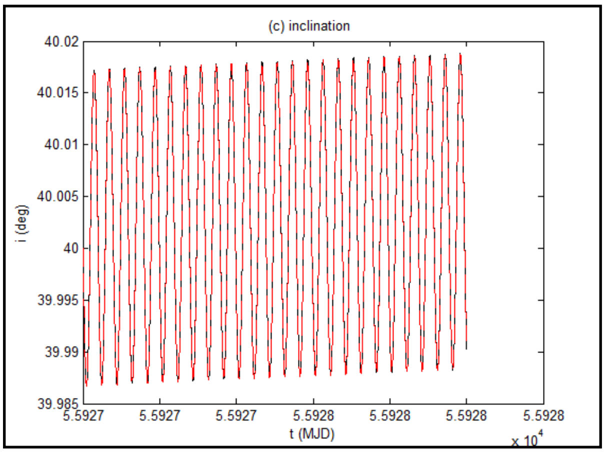
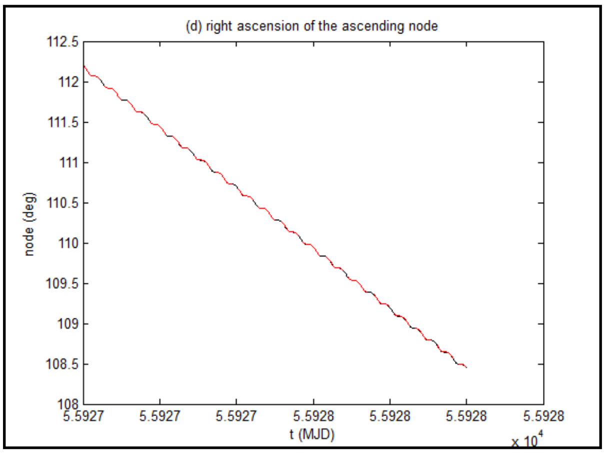
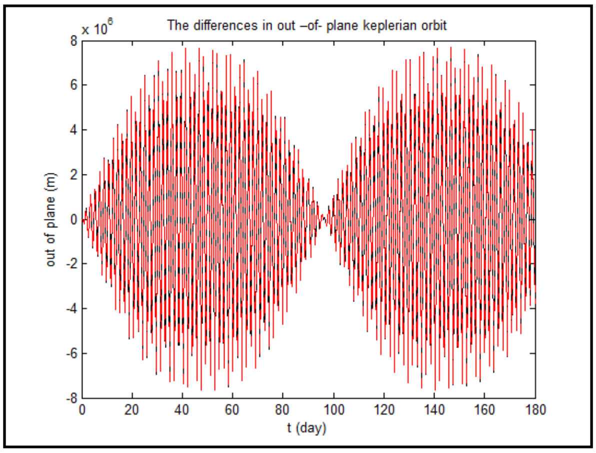


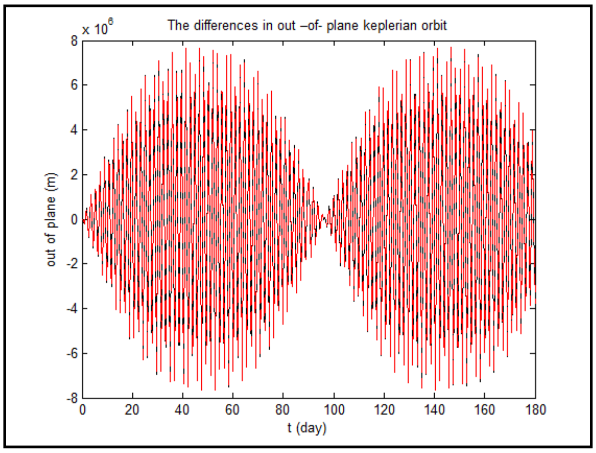
Figure (15) Variation in orbital element of a satellite due to the oblate Earth Zonal harmonic degree 50 (six month).











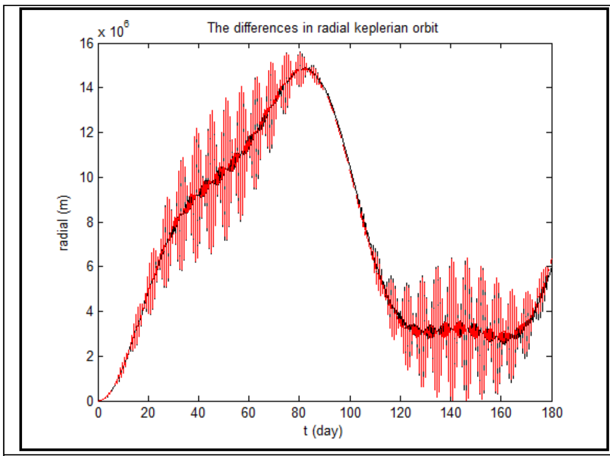
The differences w.r.t. the initial osculating Keplerian orbit (radial, along track, out of plane direction)

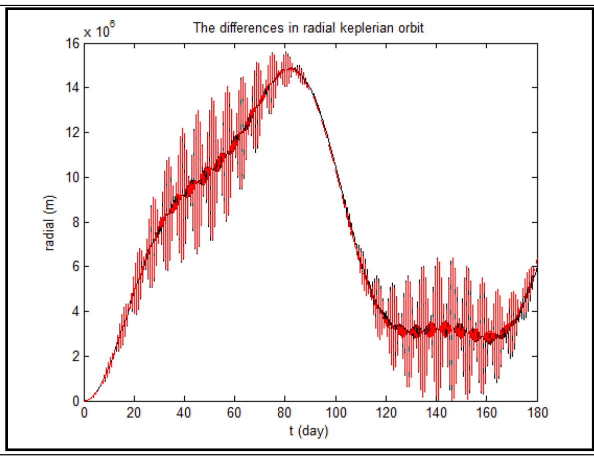
deg = 2, ord = 0 — — deg =10, ord = 0

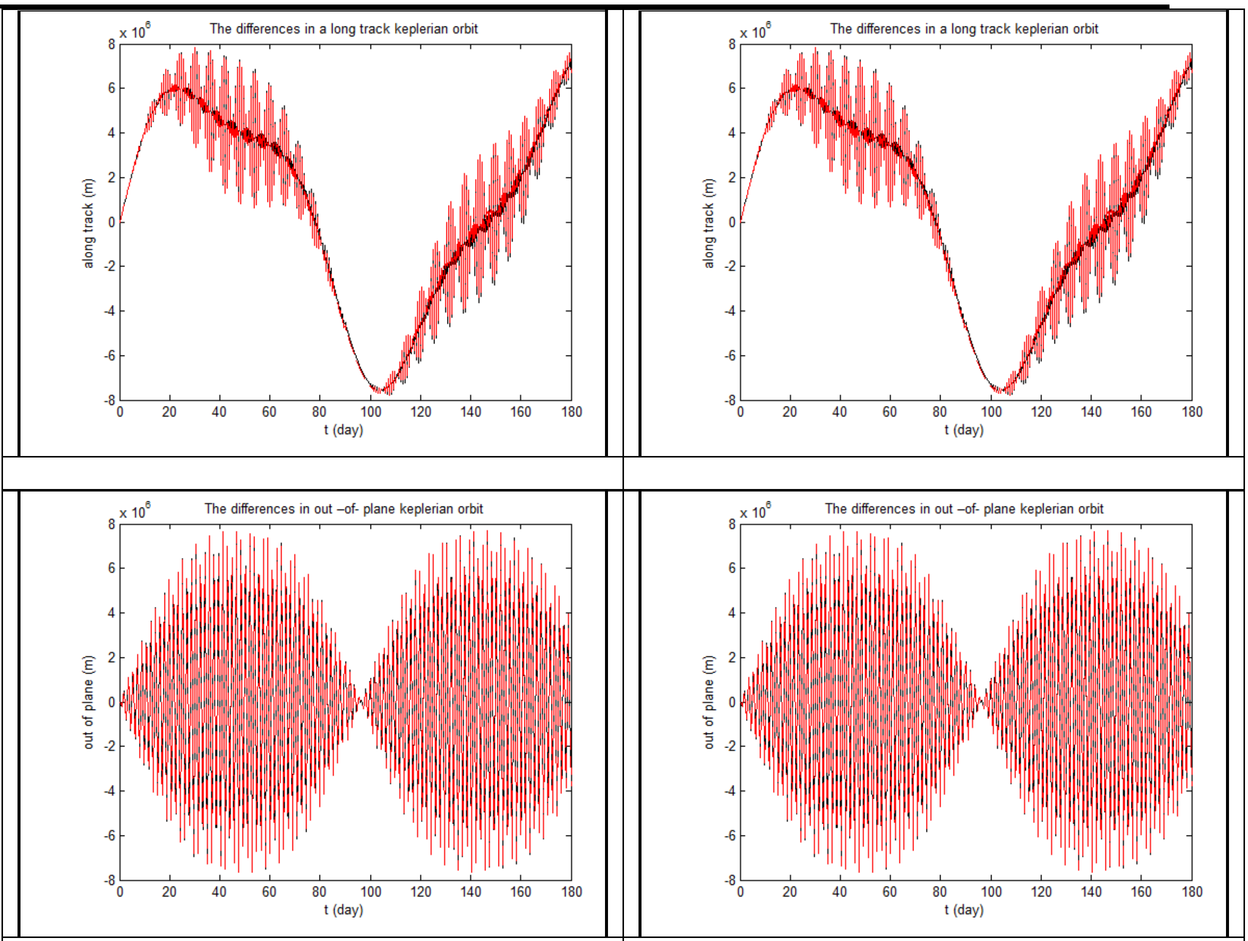
The differences w.r.t. the initial osculating Keplerian orbit (radial, along track, out of plane direction)

deg = 2, ord = 0 — — deg =4, ord = 0

Figure (16) Variation in osculating Keplerian orbit due to the oblate Earth Zonal harmonic for (six month).







The differences w.r.t. the initial osculating Keplerian orbit (radial, along track ,out of plane direction)	The differences w.r.t. the initial osculating Keplerian orbit (radial, along track, out of plane direction)
—— deg = 2, ord = 0 — — deg =50, ord = 0	—— deg = 2, ord = 0 — — deg =30, ord = 0

Figure (17) Variation in osculating Keplerian orbit due to the oblate Earth Zonal harmonic for (six month).

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